Numerical and Experimental Analysis of a Metamaterial-based Acoustic Superlens

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ABSTRACT

For many years, the engineering limitations in a single loudspeaker have offered no solution to the problem of delivering sound only to parts of an audience. Precise control on how sound is delivered to an audience has required multiple loudspeakers, either through their distribution or through DSP. The recent uptake of acoustic metamaterials, however, seem to offer different solutions. Using devices based on acoustic metamaterials, for instance, brings to acoustics design principles that come directly from optics, at a reasonable manufacturing cost. In this work, we design, numerically simulate, and characterise an acoustic converging superlens: a 3D-printed device capable of focusing an incoming plane wave at a distance less than one wavelength. We show how a loudspeaker at a fixed distance from the lens results in an “image” of the source at a distance prescribed by the thin-lens equation. Finally, we propose possible applications of such an acoustic superlens to future audio experiences.

1 Introduction

Metamaterials are subwavelength engineered materials whose properties do not depend on their chemistry but rather on their engineered geometry. The great interest in the research and development of innovative solutions that make use of this emerging technology originates mainly from their superior performance compared to traditional materials. Metamaterial-based solutions are proving to be viable alternatives to the products commonly used for noise management indoors – e.g. in hospitals [1] – or to correct the directionality (and reduce the unwanted rear emissions) in loudspeaker systems [2]. Acoustic metamaterials have found already their way into commercial audio systems, like the ones produced by KEF [3], but the problem of maximising sound intensity for an audience (or part of it) still requires electronics.

Not so in optics, metamaterials have shown that is possible to create lenses that overcome the classical laws of physics, thus enabling devices with performances not previously thought achievable. Recent studies have confirmed that it is possible to apply and prove laws which are valid in optics to acoustics as well [4] treating sound as we do with light. Researchers at the University of Sussex used this finding to demonstrate two key metamaterial-based devices inspired by optics: an auto-zoom lens, used to send sound to a moving target [4] and a diffraction grating, used to send the different notes in a melody towards multiple directions [5]. In both cases, a limited set of pre-manufactured three-dimensional units was used: metamaterial “bricks” [6], each encoding a specific phase delay on the impinging wavefront. All these studies have shown that metamaterial bricks can be assembled into metasurfaces, whose thickness is less than a wavelength, to theoretically generate any diffraction-limited acoustic field.

But diffraction becomes extremely limiting when working with lenses: the image sharpness needed to deliver sound – e.g. to a single seat in an auditorium – depends on the wavelength of sound (or light) passing through it and on its lateral dimension. This
is why many optical lens designs have been proposed that go beyond the diffraction limit, using periodic structures or metasurfaces with negative index characteristics [7].

In this work, we show that using metamaterials it is possible to go beyond diffraction also with audible sound, designing an acoustic superlens i.e., a converging lens capable of focusing a plane wave at a distance smaller than the wavelength. We compare finite elements simulations with experimental results, obtained using a 3D printed metasurface prototype dimensioned to operate near 2 kHz. We conclude discussing potential applications of this technology.

2 Superlens description

In this work, we design a metamaterial phase-plate (a “metasurface”) composed of bricks, optimised for transmission, made of a combination of 8x8 bricks, with total dimensions 216Lx27Wx215.05H [mm].

To make a converging lens, we assumed a plane wave as input to the metasurface and selected a brick distribution ensuring that all the exit wavelets would arrive simultaneously in a “focal” point. Our lens was designed for a central frequency of operation \( f_c = 2000 \text{ Hz} \) and for a focal length \( f = 150 \text{ mm} \). If the focal point is defined as the point with maximal intensity, considering sound propagation in air at a temperature of 20°C (speed of sound \( c = 343 \text{ m/s} \)), we therefore expect to find it at a distance less than one wavelength (\( \lambda = 17.1 \text{ cm} \)).

A converging “thin” lens in optics is a device whose behaviour is approximated by a limited number of parameters. The focal length, \( f \), and the distance between the centre of the lens and the source, \( p \), are connected by the “thin-lens equation”:

\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q}
\]  

(1)

where \( q \) is the distance between the centre of the lens and the image of the source. Since this equation has been proven valid also in acoustics for focal lengths greater than \( \lambda \) [4], different combinations of \( p \) and \( q \) can therefore be used to measure \( f \). Here, we will test the method for a superlens.

For wave optics, due to diffraction, the focal point spreads to become a focal spot to a dimension determined by the wavelength \( \lambda \) and by the lateral size of the lens \( D \). The Rayleigh principle prescribes that the width of the focus, represented by the angle \( \theta \) to the centre of the lens:

\[
\theta \geq \sin^{-1} \left[ 1.22 \cdot \left( \frac{\lambda / D}{\lambda} \right) \right] \quad [\text{o}] \quad (2)
\]

A lens is a superlens if the focus is smaller, in this case, than 74.98 [\text{o}], as calculated from Equation 2 using \( \lambda = 0.171 \text{ m} \) and \( D = 0.216 \text{ m} \). For the prototype realization a rectangular base of dimensions 216Lx27Wx1.5H [mm] was added to the lens to enable its stability when placed on a supporting surface for testing. The bricks selected for the prototype are cuboids of size 27Lx27Wx27H [mm]. Analytical methods described elsewhere [8] where used to determine the effective path length and thus the phase embedded to the wave passing through. To achieve the desired phase distribution, nine metamaterial bricks were eventually needed (Figure 1), with each brick different from the others in terms of the length of the internal baffles (\( l \)), the distance between baffles (\( d \)) and their number.

Figure 1. Bricks types used for the superlens design.

Figure 2 shows the designed 8x8 metasurface, together with a reference system.

Figure 2. Metamaterial-based acoustic superlens. From left to right: rear and front faces of the lens.
3 Numerical modelling

In this work, the finite element method (FEM)-based COMSOL Multiphysics 6.0 software has been utilized to perform the numerical simulations of acoustic wave propagation in the superlens. For this purpose, the standard built-in pressure acoustic module has been selected for evaluating the acoustic performance of the metastructure in the frequency domain. In order to retrieve the sound distribution around the superlens, two chambers were added to the numerical domain – i.e., one before the lens (transmitting) and one after it (receiving). Hard boundary conditions were applied to the rigid walls of the metasurface. Moreover, a perfectly matched layer (PML) was added at the boundaries of the transmitting and receiving chambers, to mimic non-reflecting infinite fields. The mesh size used for the calculation was set to be smaller than 1/6th of the shortest simulated wavelength. We used two types of sources: in the first numerical model a plane wave radiation condition \( p = \infty \) was used to simulate the incident sound waves on the rear face of the lens. Once the focal length was verified to be equal to 15 cm at the design frequency, the second simulation setup was used and the radiation condition was changed to spherical, to further study the behaviour of the lens for the example case of \( p = 30 \) cm in conditions closer to the experiments. Figure 3 shows the sound pressure level measured on a line, centred with respect to the centre of the lens, with and without the lens when \( p = 30 \) cm. This result allows to identify the \( y \)-coordinate where the largest dB-gain is expected for each frequency around \( f_o \). A maximum gain of 8.5 dB is reached around 2030 Hz at 28.3 cm.

Once the focal point was determined, as the place with the maximum gain at a fixed frequency for a fixed \( p \), the width of the focus was observed in the focal plane. Figure 4, for instance, shows the sound level pressure on a \( xz \) plane placed at \( q = 28.3 \) cm, for \( p = 30 \) cm for 2030 Hz.

The focal spot width is therefore calculated considering a 1 dB lateral decay, which corresponds approximately to 3 cm to the right and left of the centre of the focus.

4 Experimental results

The metasurface prototype has been fabricated using fused deposition 3D printing of a polylactide (PLA) 1.75 mm diameter filament. The material chosen for the prototype realization is completely biodegradable. The printer used guaranteed a 0.4 mm diameter print nozzle and a layer height of 0.2 mm. The source was mounted on a pedestal, so that 132 cm could be measured between the centre of a 3-inch (7.6 cm) woofer (Sealed box enclosure, extended range: 100 - 10k Hz, model: 3FE25 Faital) and the floor. A second pedestal equipped with a support surface was used for the lens, so that the centre of the lens was in line with the acoustic centre of the sound source. An omnidirectional microphone (\( \frac{1}{4} \)-inch Earthworks M30) was placed at the same height as the sound and the superlens thanks to a dedicated tripod, allowing its position to be varied in the horizontal plane \( xy \). No measurements were made in the vertical plane. The signal emitted is a LogChirp from 10 Hz to 22 kHz, repeated three times. The signal is acquired with a CLIO data acquisition board and processed with an Half-Hanning window, having \( f_o = 48 \) kHz as sampling frequency \((N = 16k \) samples).
In a locally acoustically treated measurement laboratory, we repeated a first set of measurements to confirm Equation 1, thus searching for the sound image $q$ at different distances between the source and the superlens $p$ following the set-up schematically illustrated in the schematic Figure 5.

Figure 5. Set-up schematic view (yz plane) used to measure $f$ using the thin-lens equation.

This set of measures allowed us to demonstrate that, around the design frequency of the lens, the thin-lens equation works also for a metamaterial-based acoustic superlens. Figure 6 reports the measured results and both horizontal and vertical error bars, considering a 10% range. The measured value of $f_{\text{meas}} = 144 \pm 12$ mm is compatible with the design value.

Figure 6. On-axis measured results at 2 kHz.

Figure 7 shows the on-axis frequency response measured at different $q$ when $p= 30$ cm, between 1.7 kHz and 2.2 kHz. Note that the curves in Figure 7 have been normalized with respect to the sound pressure level measured in the same position but without the lens, to highlight the dB-gain reached at the sound image. This type of measurements allows to determine what happens when the frequency is close to the design one. In this configuration, Equation 1 places the image at $277 \pm 23$ mm, but the maximum gain of 5.5 dB is reached at the frequency of 1935 Hz, which is very close (3%) to the design frequency of the superlens. Previous studies [8] have shown that the thermo-viscous effects are negligible in this design, so the difference is attributed to the manufacturing. The difference between the maximum gain of 5.5 dB and the simulated one of 8.5 dB can be attributed to the simplifications in the numerical model, introduced to reduce its calculation weight.

Figure 7. Normalized frequency response for the case $p= 30$ cm at different $q$.

Once the focus has been identified, measurements along the $x$-direction were performed at the distance of maximum gain, without varying the $y$-coordinate. The measured results showed that a consistent decay ($\Delta = 1$ dB) occurs after 5.4 cm with respect to the centre of the lens, both on the right and left sides. Thus, we fitted our emission with a Gaussian profile. In this way, we estimated that 68% of the emission is within a total angle $\theta = 60.4 \pm 5^\circ$. Since this value is smaller than the limit given by the Rayleigh principle, this confirms that our lens is a superlens, according to Equation 2.

5 Future applications

The performance of the superlens designed in this work is limited in bandwidth: the maximum dB-gain is reached only within $\pm 10\%$ of the central frequency. This was intentional, as the main purpose of this work is a proof-of-concept.

Once the bandwidth is sufficiently extended – e.g. 2.5 octaves have been measured for some of the prototypes in [4] – such a lens could be coupled to different types of transducers for both emission and measurement of sound pressure. On the other hand, a superlens can be thought as a complementary solution...
to digital signal processing in loudspeaker systems that are equipped with DSP, thus extending the control of directivity on both horizontal and vertical planes or limiting destructive interference due to the possible interactions with obstacles in space. Nowadays, the continuously improving manufacturing techniques for metastructures fabrication, has made it possible to produce metamaterial-based devices that are less bulky, lighter and sometimes transparent for light than those built in the past [1,2]. Thanks to these devices, integrated into traditional audio reproduction systems, it will be possible to achieve special effects for sound thus, limiting the constrains of traditional solutions. However, passive metasurfaces have limitations too because once you have designed one that is what you have. Thus, hybrid systems should be designed, complementing mechanical movement of a system of metamaterial-based acoustic lenses [4] with real-time DSP. Moreover, superlenses could find an application in recording for audio monitoring (near or mid field monitors) or they could integrated efficiently in a listening environment to reduce unwanted reflections from surfaces (e.g. desk, mixer, walls) at specific frequencies. Systems of metamaterial-based devices, such as the one designed in this work, could also be employed for room sound reinforcement since they achieve high directivity in both horizontal and vertical planes when desired, unlike classical loudspeaker arrays.

6 Conclusions
This work demonstrates with both numerical simulations and experimental measurements that it is possible to design and realise an acoustic superlens working at audible frequencies. It also shows, with simulations and measurements, that the thin-lens equation could also be applied to such a superlens, with more statistical tests (e.g., with different superlenses) necessary before a general conclusion on the validity of this design tool is reached. The mismatch between measurements and simulations is largely consistent with errors due to measurement and manufacturing uncertainties. For future measurement tests, a more accurate mapping of the field should be obtained, both in the horizontal and vertical planes at the output of the superlens e.g., by mounting the microphone on a mechanised system.

References