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On the efficiency of flown vs. ground-stacked subwoofer configurations

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ABSTRACT

Modern live loudspeaker systems consist of broadband sources, often using variable curvature line sources, combined with subwoofers. While it is common practice to fly the broadband sources to improve energy distribution in the audience, most subwoofer configurations remain ground-stacked because of practical constraints and alleged efficiency loss of flown configurations.

This article aims at evaluating the efficiency of flown subwoofers for large audiences, as compared to their ground-stacked counterparts. We use finite element simulations to determine the influence of several factors: baffling effect, trim height. We show that flown configurations remain efficient at the back of the venue while reducing the SPL excess at the front of the audience.

1 Introduction

Large scale loudspeaker systems often consist of variable curvature line sources (VCLS) and multiple subwoofers. If VCLS are flown and properly designed, they offer a good compromise between frequency response stability and SPL uniformity. For a 100 m deep venue, less than 10 dB front-to-back SPL difference can be achieved in the 1-10 kHz range [1].

To maintain a good frequency response stability when adding a subwoofer system, the latter should fulfil two conditions:

- a front-to-back SPL similar to the main system
- a phase-coherent summation with the main system throughout the audience

These conditions are difficult to achieve with ground-stacked subwoofers since:

- The front-to-back SPL can be extreme (over 30 dB for a 100 m deep venue) with an excessive level in the front.
- The distance difference, hence the relative phase, between the subwoofers and the flown VCLS depends on the position in the audience and results in a compromised alignment.

On the contrary, flown subwoofer configurations offer:

- a moderated front-to-back SPL, by reducing the excess of level at the front of the audience
- an enhanced summation in the low frequency range throughout the audience, by being closely coupled to the main source

However, flown subwoofer configurations are often disregarded for some alleged efficiency issues as compared to ground-stacked configurations.

The common belief is that flown subwoofers are 6 dB less efficient because they lost the floor coupling effect. In reality, the floor is still present and a flown subwoofer will only exhibit a 3 dB reduction in the total radiated power, due to the separation from the floor. However, one needs to assess the SPL efficiency where SPL is needed: over the audience.

A very simple measurement can support this observation and justify the need to revisit the common belief.

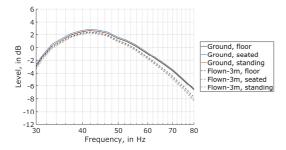


Figure 1: Measured frequency response of a subwoofer (L-Acoustics KS28¹) on the ground or flown at 3 m height, 20 m distance; microphone on floor, seated height (1.2 m), standing height (1.6 m)

In Figure 1, we show the frequency response of a single L-Acoustics KS28 subwoofer in its operating frequency range, measured at 20 m for two deployments (ground-stacked, flown at 3 m) and for typical listening heights. We observe that the level differences are negligible, far from the alleged 6 dB loss.

In this paper, we examine different design factors that affect the performance of subwoofer configurations, such as the type of arrangement, the number of subwoofers and the distance from ground for flown configurations. We provide a framework for comparing various subwoofer configurations.

We consider a simple environment with a rigid floor as the only reflective surface. These conditions model open-air situations and simplified large indoor venues.

The influence of the audience being hardly predictable, adding little to no absorption at low frequencies is neglected here [3].

We describe first the coupling of subwoofers with ground, using a simple image source model [4] and omnidirectional point sources. We demonstrate that there is a listening distance starting from which there is negligible level difference between flown and ground-stacked subwoofers. This minimum listening distance depends on subwoofer and listening height. Flown subwoofer configurations therefore maintain the same far field efficiency while avoiding the SPL excess at the front of the audience.

We then consider a complex Finite Element Model (FEM) including all effects related to the cabinet diffraction [5][6]. We test various configurations of subwoofers at a large distance (100 m). Observed level differences between ground-stacked and flown (0.15 to 8 m) configurations are below 0.5 dB for a single subwoofer and a vertical array of 4 subwoofers, and below 1 dB for a horizontal array of 4 subwoofers.

In a second part, we compare the efficiency of various subwoofer configurations considering an audience area that is scaled to the number of subwoofers used. We introduce two evaluation criteria that describe the far field SPL efficiency and the distribution of SPL over the audience.

We first examine flown vertical arrays of subwoofers. We show that they offer optimum efficiency, independently of their height from the floor, and a homogeneous SPL distribution.

We then compare a flown central configuration with other standard subwoofer configurations (different flavours of left-right arrangements and ground-stacked horizontal arrays: arc subwoofer). We show that the flown central configuration is on average 4 dB more efficient than the other configurations, while offering an optimum SPL distribution over the audience.

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¹ http://www.l-acoustics.com/products-ks28-148.html

2 Subwoofer coupling with ground

2.1 Mirror effect

We consider the simple case of a single omnidirectional subwoofer in the presence of a rigid flat infinite floor. A simple image source predicts two contributions:

- direct sound
- floor reflection

We model the SPL at the ground level first, then at standard listener height (seated or standing).

2.1.1 Ground level

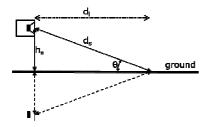


Figure 2: Contributions of direct and mirrored sources at the ground level

In the case of a single subwoofer modeled as an omnidirectional point source, the level at the ground level is simply obtained as:

$$L_g = 2\frac{A}{d_I} \tag{1.1}$$

for a ground stacked subwoofer and

$$L_f = 2\frac{A}{d_s} \tag{1.2}$$

for a flown subwoofer, where A is the level at 1 m distance of a single subwoofer in free field.

The resulting SPL difference between a ground-stacked and a flown subwoofer at the distance d_l is:

$$L_{r,g} = \frac{L_f}{L_g} = \cos\theta \tag{1.3}$$

Where:

$$\theta = \arctan\left(\frac{h_s}{d_I}\right) (1.4)$$

 θ tends to 0 for large d_l . Therefore, $L_{r,g}$ tends to 1 for large d_l , meaning there is no level difference at large distances between the flown and ground-stacked deployment.

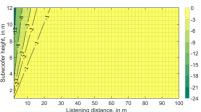


Figure 3: Level reduction at ground level $L_{r,g}$ of flown vs. ground subwoofer

Figure 3 represents the observed $L_{r,g}$ depending on subwoofer height and listening distance. We observe that a significant level reduction is only observed at small listening distances where the SPL from ground-stacked subwoofer is excessive.

We can define a minimum listening distance at which a flown subwoofer is less than 1 dB than the ground stacked configuration. This distance linearly depends on subwoofer height h_s .

$$d_{l,1dB} = h_s / \tan \left(\arccos \left(10^{\left(\frac{-1}{20} \right)} \right) \right)$$
 (1.5)

which simplifies to:

$$d_{L1dB} \approx 2 * h_s \qquad (1.6)$$

2.1.2 Ear level

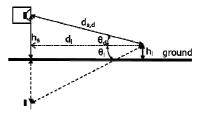


Figure 4: Contributions of direct and mirrored sources at an arbitrary listening height

As can be observed in Figure 4, the distances $d_{s,d}$ and $d_{s,i}$ of direct and image contributions to the listening point are different.

Therefore, L_r is depending on the frequency and can be expressed as:

$$L_{r}(d_{l}, h_{l}, \omega) = \frac{\left(\cos \theta_{d} + \cos \theta_{l} * \cos \left(\frac{\omega}{c} * \left| \frac{d_{l}}{\cos \theta_{l}} - \frac{d_{l}}{\cos \theta_{d}} \right| \right)\right)}{2} (1.7)$$

The general tendency shows that L_r tends to 1 for large distances d_l . In Figure 5, we show the level reduction obtained at standing level (1.6 m), integrating energy between 30 and 80 Hz.

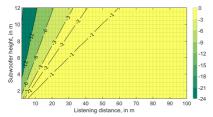


Figure 5: Level reduction of flown vs. ground subwoofer, 30 to 80 Hz, standing level (1.6 m)

2.1.3 Design guidelines

We define the distance to height efficiency ratio *DHER* as the ratio between $d_{l, IdB}$ and the subwoofer height h_s . We find that:

Ear level	Ground	Seated	Standing
DHER	2	4	5

For any other listening height, DHER can obtained with the following formula:

$$DEHR \approx 2.7 * h_1 + 0.75$$
, for $h_e > 0.9$ m (1.8)

DHER can be used in two ways, providing either a minimum distance at which the full efficiency of the subwoofer configuration is obtained for a given subwoofer height or as the maximum height at which the subwoofer should be flown for a given venue depth.

For example, in a 50 m deep venue with standing audience, the subwoofer can be flown up to 10 m without losing efficiency at the back. In that case, it leads to a better SPL distribution, with the front-to-back SPL reduced by more than 12 dB, compared to the ground-stacked subwoofer.

2.2 Baffling effect

2.2.1 Description

Subwoofers are often described in the literature as omni-directional sources assuming that the dimensions of their cabinet are small, compared to the wavelength (from 4.3 m to 11.4 m within the defined bandwidth: 30 to 80 Hz). This assumption might hold true for consumer devices, but it is only partly correct for professional subwoofers.

As an example, the KS28 subwoofer of L-Acoustics is a double 18 inches subwoofer that measures 1.34*0.7*0.55 m (width, depth, height). It is often arrayed in multiple units forming a large baffle.

The effect of the loudspeaker enclosure has been experimentally observed for different enclosure shapes by Olson in [5].

In the case of a large sphere, Olson could observe an increase of 6 dB from low to high frequencies that corresponds to the transition of radiating sound in a 4π space to a 2π space, the sphere acting as an infinite baffle at high frequencies. Loudspeaker cabinets have a more complex diffraction pattern because of sharp edges.

Urban et al. proposed a simple model of cabinet edge diffraction in [6]. The Dipole Edge Diffraction (DED) model describes the frontal baffle edges as dipolar secondary sources that interfere with the primary radiating elements of the loudspeaker (membranes, vents). The DED model highlights the impact of the size and shape of the frontal baffle on the loudspeaker radiation.

2.2.2 Simulations

In the case of ground-stacked subwoofers, the direct and mirrored sources create a continuous baffle that has twice the size of the original configuration along the direction perpendicular to the ground. In the case of flown subwoofers, the direct and the mirrored sources are separated by a gap that increases with subwoofer height. Three configurations were tested (see Figure 6). The subwoofers are placed in a simple environment where subwoofer height varies over a pseudo-infinite flat rigid and floor, as shown in Figure 6. The pressure is calculated using the FEM solver of Comsol® Multiphysics at 100 m distance from the subwoofers at ground and standing level. We use an extensive model of the L-Acoustics KS28 subwoofer that simulates the full speaker-enclosure interaction.



Figure 6: Subwoofer array configurations for baffling effect simulation (left: single subwoofer, 1Sub; center: vertical array, 4Vert; right: horizontal array, 4Hori)

In Figure 7, we show the level differences at 100 m between the flown and the ground-stacked configurations at multiple subwoofer heights. Spreading loss induced level differences are compensated for using equation (1.7).

The largest level differences are observed for the horizontal array of 4 subwoofers but remain below 1 dB. The level differences are similar on the ground or standing height (1.6 m) and that half of the level differences already appear for $h_s = 0.15$ m.

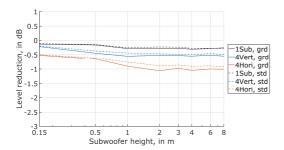


Figure 7: Simulated level differences (30-80 Hz) between ground-stacked and flown subwoofer array configurations, 100 m distance, at ground level (grd) and standing level (std)

In a second set of simulations, we use the DED model and a simple image source model to model the presence of the ground. Loudspeakers are described as rigid disks of 18 inches diameter.

Figure 8 shows results the level reduction observed at 8 m height for the DED and the full FEM model. The DED model tends to overestimate the baffle effect but follows the same tendencies. We observe that the baffle effect is negligible for vertical arrays of more than 8 subwoofers.

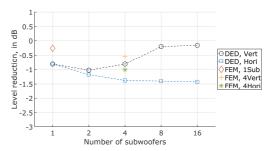


Figure 8: Baffling effect at 8 m height, as predicted by the DED model against FEM simulation

3 Configuration benchmarking

In this part, we first introduce evaluation metrics that aim at better describing the performance of subwoofer configurations over an extended listening area. We concentrate then on configurations of vertical arrays of subwoofers and evaluate the influence of the number of subwoofers and their height. Finally, we compare typical ground-stacked and flown configurations.

3.1 Evaluation framework

3.1.1 Test configuration

We consider configurations that double the number of subwoofers n_s at each step: 2, 4, 8, 16. Therefore, we may anticipate an ideal 6 dB increase of efficiency at each step, for each doubling of n_s .

We consider an audience area that scales linearly according to $log_2(n_s)$:

- 20, 40, 60, 80 m in length
- 15, 30, 45, 60 m in width
- with 0.5 m resolution in both directions

Independently of n_s , the audience area starts 2 m away from the front of the subwoofers.

The pressure in the audience at standing height (1.6 m) is simulated using the DED model of the KS28 subwoofer, as described in section 2.2.2, using a 12th octave frequency resolution from 30 to 80 Hz.

The DED model was chosen for its simplicity and efficiency to simulate many configurations for up to 19200 listening positions.

3.1.2 Evaluation metrics

The far-field SPL efficiency criterion *L-Eff* is calculated as the difference between the average banded SPL obtained for the test configuration and the average banded SPL for one ground-stacked subwoofer.

$$L-Eff_{t} = \langle SPL_{t}(r) \rangle_{rear} - \langle SPL_{1sub}(r) \rangle_{rear}$$
(1.9)

Banded SPL (30-80 Hz) is averaged over the rear (last quarter of audience), as represented in Figure 9. This criterion provides an estimate of "return on investment" of adding subwoofers in terms of far field SPL. The larger the value, the better.

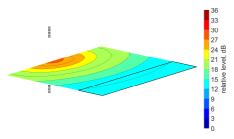


Figure 9: SPL mapping for a vertical array of 4 subwoofers relative to the SPL of 1 ground-stacked subwoofer averaged in the rear (black rectangle)

The second criterion *L-Dis* is calculated as the 95% interval against the median SPL over the entire audience area. It represents the distribution of SPL. Small values (below 10 dB) indicate a homogeneous SPL distribution. Larger values indicate an excess of SPL in portions of the audience, typically in front. The *L-Dis* criterion is more representative of the SPL differences experienced by all listeners than the *audience and FOH level consistency* proposed in [7]. The later compares the level at a critical, but single, position against the average SPL over the

entire audience and therefore neglects close listening positions.

3.2 Flown vertical arrays

3.2.1 Test configuration

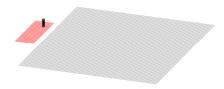


Figure 10: Center flown vertical subwoofer configuration (black square) with stage (red surface) and audience (grey surface).

We consider a vertical array with the values of n_s of section 3.1.1. The subwoofer height h_s variation depends on n_s (from 0 to 3, 4.6, 6.3 and 8 m) to account for typical installation constraints.

3.2.2 Simulation results

Figure 11 shows L-Eff depending on depending on n_s and h_s as well as the expected gain of efficiency obtained for each doubling of subwoofer number. We observe that L-Eff is above the expected gain in efficiency which indicates a more than optimum return on investment of the vertical array configuration. The excess of gained efficiency can be attributed to the baffling effect for longer lines.

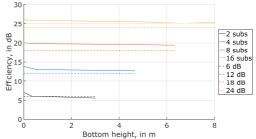


Figure 11: *L-Eff* for the center flown subwoofer configuration depending on n_s and h_s

Figure 12 shows L-Dis depending on n_s and h_s . We observe that L-Dis decreases with h_s and n_s and reaches values below 10 dB for any number of subwoofers in flown configurations.

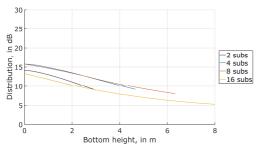


Figure 12: *L-Dis* for the center flown subwoofer configuration depending on n_s and h_s .

3.2.3 Discussion

This simulation shows that flying vertical subwoofer arrays is beneficial in terms of distribution and remains efficient, independently of subwoofer height. It also shows that the DHER criterion defined in section 2.1.2 is valid for arrays of up to 16 elements (8.8 m long) where h_s is the bottom height of the subwoofer array.

Longer arrays should be preferred over shorter ones to minimize the SPL distribution. Long arrays are however difficult to install because of ceiling height and requested visual clearance. The solution could be to split them into two smaller arrays, deployed in an inline arrangement with end-fire processing [7] or side-by-side.

3.3 Configurations comparison

3.3.1 Simulation framework

We compare typical subwoofer configurations with a varying n_s as described in section 3.1.1. We consider 5 configurations:

- flown central vertical array (Flown C): h_s = 3, 4.6, 6.3, 8 m for n_s =2, 4, 8, 16
- flown Left-Right vertical arrays (Flown LR): h_s = 1.9, 2.9, 4, 5 m for n_s =2, 4, 8, 16
- ground-stacked Left-Right (Ground LR): 2 compact arrays on either side of the stage.
- ground-stacked horizontal array, no spacing between subwoofers (Arc Compact).
- ground-stacked horizontal array, stacks of one subwoofer spanning all stage width (Arc Wide).

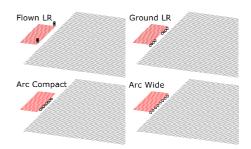


Figure 13: Subwoofer configurations used for n_s =8.

We consider a scalable stage width of 8, 12, 16, 20 m for n_s =2, 4, 8, 16 respectively. The Flown C configuration is displayed in Figure 10. The other configurations are displayed in Figure 13.

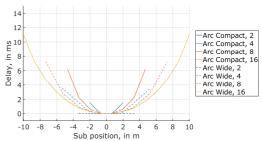


Figure 14: Delays used for arc subwoofer configurations.

Delays used for the arc subwoofer configurations are displayed in Figure 14. They were optimized to jointly maximize *L-Eff* and minimize *L-Dis* independently for each configuration.

3.3.2 Simulation results

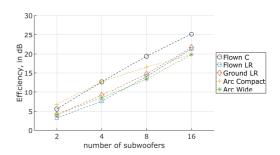


Figure 15: *L-Eff* for tested subwoofer configurations calculated and different number of subwoofers.

Figure 15 shows *L-Eff* for the 5 typical arrangements depending on n_s . We see that the central flown configurations are overall the most efficient.

To refine our analysis, we show in Figure 16 the efficiency relative to the Ground LR configuration. We see that the Arc Compact configuration remains efficient for up to 8 subwoofers. For 16 subwoofers however, the Arc Compact and Arc Wide configurations are identical because the KS-28 subwoofer is 1.34 m wide.

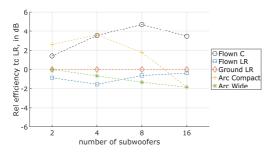


Figure 16: efficiency relative to ground LR configuration.

The Arc Wide configuration is 6 dB less efficient than the Flown C configuration for 8 and 16 subwoofers. To reach the same far field SPL with the Arc Wide, a double number of subwoofers is therefore required, by doubling the height of each stack rather than increasing the length of the array. The Flown LR configuration is 0.5 to 1.5 dB less efficient than the Ground-stacked LR. The Flown LR provides the same average efficiency as the Arc Wide but is more efficient by 0.5 to 1 dB for 8 and 16 subwoofers.

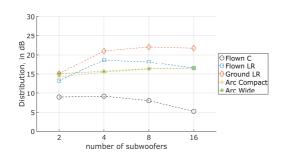


Figure 17: *L-Dis* for typical subwoofer configurations calculated over the scalable audience area.

Figure 17 shows L-Dis for the 5 typical arrangements depending on n_s . We observe that L-Dis is the lowest for the Flown C. It is 5 to 12 dB lower than any other configuration. The Arc Compact and Arc Wide are not significantly different. They are on average 5 dB better in L-Dis compared to Ground LR.

The Flown LR performs similarly to the Arc Compact and Arc Wide configurations for 2 and 16 subwoofers. It remains better than Ground LR for 4 and 8 subwoofers but does not perform as well as the Arc Compact and Arc Wide configurations.

3.3.3 Discussion

Overall, the Flown C configuration is the best, both in terms of efficiency and distribution.

Ground-stacked configurations require up to twice the number of subwoofers to achieve the same far field SPL and never reach the same distribution.

The SPL mapping relative to the average SPL in the rear part of the audience is displayed from Figure 18 to Figure 22. We see that the most homogeneous is the Flown C configuration.

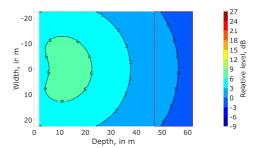


Figure 18: Flown C, n_s =8, h_s =6.3 m

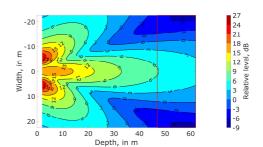


Figure 19: Ground LR, n_s =8

Ground and Flown LR exhibit the well-known power alley of Left-Right arrangements.

The advantages of Flown LR are:

- the limited excess of level in the front of the audience.
- its physical proximity to the VCLS in a typical LR arrangement that allows for a best summation throughout the audience.

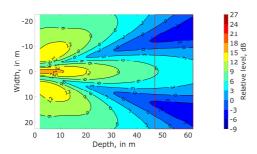


Figure 20: Flown LR, n_s =8, h_s =4 m height

The Arc Wide and Compact configurations can achieve a very similar SPL pattern. We may conclude that overall the Arc Compact should be preferred over the Arc Wide configuration since it is up to 4 dB more efficient.

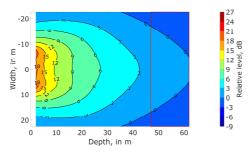


Figure 21: Arc Wide, n_s =8

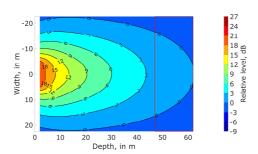


Figure 22: Arc Compact, n_s=8

4 Conclusions

The objective of this paper is to revisit the belief that flown subwoofer configurations are less efficient than their ground-stacked counterparts.

We re-establish what efficiency is, the necessity to assess SPL performances over the audience and introduce adapted evaluation criteria.

The absolute SPL efficiency should be evaluated in terms of absolute throw capability, i.e. as the SPL at the rear of the audience. This is where the direct sound SPL reaches its minimal values due to propagation loss. We therefore introduce the far-field SPL efficiency as the first criterion.

The complementary criterion we propose is the SPL distribution, closely linked to the front-to-back SPL in a venue, or relative throw capability of the source. For a same far-field efficiency, the discriminating factor is the homogeneity of the SPL distribution over the audience. Having less SPL at the front of the audience is preferable, and even desirable, to preserve auditory health. In that regards, the legislation is evolving in many countries to limit the maximum SPL(C) [9]. Ground-stacked subwoofers lead to excessive SPL(C) in front. Therefore, flying subwoofers is a rational solution despite the well-known challenges in implementation.

We can also support the flown solutions by observing their pure SPL performances.

In the first part of this article, we explore the impact of decoupling the subwoofer from the floor in relation to the mirror and baffle effects.

It is shown that a flown subwoofer have a similar far-field efficiency to that of a ground-stacked subwoofer when a maximum subwoofer height is respected. This maximal height is linked to the venue depth via the DHER criterion, and depends on the listening height. For a standing audience, SPL efficiency is recovered for listening distances that are 5 times or more the subwoofer height. The audience benefits from a more homogeneous SPL distribution and an important SPL reduction close to the stage.

When looking at the loss that could arise from the split of the baffle size with source separated from the floor, we show that it is mostly negligible and almost inexistent for a vertical array of 8 or more 18 inches subwoofers.

In the second part of the article, we proceed to a configuration benchmark on standard audience areas, first among different variations of central vertical arrays, and then by comparing different standard subwoofer system configurations.

We observe a negligible reduction of far-field SPL efficiency between a ground-stacked central array and the same flown array that respects the DHER, extending the validity of this criterion from a single subwoofer to subwoofer arrays. When increasing the array size, we show that the baffle effect leads to more SPL than simply adding the contribution of each array element. This remains valid for any subwoofer height.

The last results demonstrate that central flown configuration is the optimal solution. It is the only one that always remains below 10 dB of SPL distribution and is then able to keep on with the HF SPL distribution provided by variable curvature line sources. It is also the most efficient, giving twice as much SPL as flown LR, ground-stacked LR or arc wide configurations at the back of a 60 m deep venue using a total of 8 subwoofers.

Left-right configurations suffer from their well-known interference pattern, whether flown or ground-stacked, but the ground-stacked LR gives the worst SPL distribution, about 20 dB on average.

The arc wide configuration, although one of the most common deployment nowadays, is the less efficient and one should consider to rather adopt an arc compact if ground-stacking is the only deployment option.

Finally, we need to consider the alignment of the subwoofer system with the main. The relative phase variation over the audience is directly linked to the spacing between both systems. With a standard LR flown main, LR flown subwoofers are obviously the solution that offers the best coupling, despite the fact it can reinforce the power alley. Both the LR ground-stacked subwoofers and the C flown subwoofers are decoupled from the LR main in either the vertical or horizontal dimension. The arc sub solutions are decoupled in both dimensions and are always a challenge to align.

Multichannel solutions that use multiple main sources above the stage are becoming increasingly popular in the live sound industry. In this case, the

central flown subwoofer configuration is the obvious choice because it provides a close coupling with several of the main sources and offers the best SPL far-field efficiency and distribution.

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