# Wavefront Sculpture Technology*

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The Fresnel approach in optics is introduced to the field of acoustics. Fresnel analysis provides an effective, intuitive way of understanding complex interference phenomena and allows for the definition of criteria required to couple discrete sound sources effectively and to achieve coverage of a given audience geometry in sound-reinforcement applications. The derived criteria form the basis of what is termed Wavefront Sculpture Technology.

## 0 INTRODUCTION

This paper is a continuation of the preprint presented in 1992 at the 92nd Convention of the Audio Engineering Society [1]. Revisiting the conclusions of that study, which were based on mathematical analysis and numerical methods a more qualitative approach based on Fresnel analysis is presented, which enables a better understanding of the physical phenomena involved in arraying discrete sound sources. From this analysis, the criteria that define how an array of discrete sound sources can be assembled to approximate a continuous line source are established. When considering a flat line source array, these criteria turn out to be the same as those that were originally developed in [1]. A variable-curvature line source array is also considered, and further criteria are defined in order to produce a wavefield that is free of destructive interference over a predefined coverage region as well as a wavefield intensity that decreases as the inverse of the distance over the audience area. These collective criteria are termed Wavefront Sculpture Technology (WST) criteria.[1]

## 1 MULTIPLE-SOUND-SOURCE RADIATION — A REVIEW

The need for more sound power to cover large audience areas in sound-reinforcement applications implies the use of multiple sound sources. A common practice is to configure loudspeakers in arrays or clusters in order to achieve the required sound pressure level (SPL). Typically, trapezoidal horn-loaded loudspeakers are assembled in fan-shaped arrays according to the angles determined by the nominal horizontal and vertical coverage angles of each enclosure in an attempt to reduce coverage overlap, which results in destructive interference. However, since the directivity of the individual loudspeakers varies with frequency, the sound waves radiated by the arrayed loudspeakers do not couple coherently, resulting in interference that changes with both frequency and listener position.

Considering early line array systems (column loudspeakers), apart from narrowing of the vertical directivity, another problem is the appearance of secondary sidelobes, outside the main beamwidth, whose SPL can be as high as the on-axis level. This can be improved with various tapering or shading schemes; however, the main drawback is a reduced SPL. For the case of Bessel weighting, it was shown that the optimum number of sources is five [2], and this is considered insufficient to meet modern sound-reinforcement requirements.

In [1] the use of line source arrays for sound-reinforcement applications was presented, where the objective is to produce a wavefront that is as continuous as possible. Considering first a flat, continuous, and isophasic (constant-phase) line source, it was demonstrated that the radiated sound field exhibits two spatially distinct regions: the near field and the far field. In the near field, wavefronts propagate with 3-dB attenuation per doubling of distance (cylindrical wave propagation) whereas in the far field there is 6-dB attenuation per doubling of distance (spherical wave propagation). It is to be noted that parameters such as directivity and polar plots cannot be considered in the near field (as developed in Appendix 1).

Considering a line source with discontinuities, a progressively chaotic behavior of the sound field was observed as these discontinuities became progressively larger [1]. This was confirmed in [3] for an array of 23 loudspeakers, where 7-dB SPL variations over 1 ft (vertically) were observed in the near field. Raised cosine

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weighting was applied in order to reduce these variations and was somewhat successful, but it is not possible to have, at the same time, raised cosine weighting for the near field and Bessel weighting for the far field. In [1] it was shown that another way to minimize these effects is to build a quasi-continuous line source by arraying discrete sound sources in accordance with a certain number of criteria. A line array of discrete sources that respects these criteria and successfully approximates a continuous line source is termed a line source array.

The location of the border between the near field and the far field is a key parameter that describes the wavefield radiated by a line source array. Fig. 1 displays a vertical section view of the radiated sound field, where the SPL is significant only in the shaded region (ABCD and the cone beyond BC). Considering a flat, continuous line source of height $H$ that is radiating a flat, isophasic wavefront, it was demonstrated in [1] that a reasonable average of the different possible expressions for the border distance $d_B$ that are obtained by using either geometric, numerical, or

Fresnel calculations is

$$d_B = \frac{3}{2} FH^2 \sqrt{1 - \frac{1}{(3FH)^2}}$$

where $d_B$ is the distance in meters from the array to the border between the near and far field regions, $F$ is the frequency (in kHz), and $H$ is the height of the line source (in meters).

There are three things to note about this formula:

1) The root factor indicates that there is no near field for frequencies lower than $1/3H$. Hence a 4-m-high array will radiate immediately in the far-field mode for frequencies less than 80 Hz.

2) For frequencies above $1/3H$ the near-field extension is approximately linear with frequency.

3) The dependence on the dimension $H$ of the array is not linear but quadratic.

As a result, the near field can extend quite far away at high frequencies. For example, Fig. 2 illustrates the vari-



Fig. 1. Radiated SPL for line source AD of height $H$. In the near field, SPL decreases as 3 dB per doubling of distance whereas in the far field, SPL decreases as 6 dB per doubling of distance.



Fig. 2. Variation of border distance and far-field coverage angle with frequency for a flat line source array of 5.4-m height.

ation of border distance and far-field coverage angle with frequency for a flat, continuous line source of 5.4-m height. For this example, the near field extends as far as 88 m at 2 kHz.

It should be noted that different authors have proposed various expressions for the border distance:

- $d_B = 3H$           Smith [3]
- $d_B = H/\pi$           Rathe [4]
- $d_B = \max (H, \lambda/6)$    Beranek [5]

Most of these expressions omit the frequency dependence and are incorrect with regard to the size dependence.

It was also demonstrated in [1] that a line array of sources, each radiating a flat isophase wavefront, will produce secondary lobes not greater than $-13.5$ dB with respect to the main lobe in the far field and SPL variations not greater than $\pm 3$ dB within the near-field region, provided that:

- Either the sum of the flat, individual radiating areas covers more than 80% of the vertical frame of the array, that is, the target radiating area,[2] or
- The spacing between the acoustic centers of individual sound sources is smaller than $1/6F$ that is, $\lambda/2$, given the approximation that $\lambda = 1/3F$, where $\lambda$ is the wavelength in meters and $F$ is the frequency in kHz.

These two requirements form the basis of WST criteria which, in turn, define conditions for the effective coupling of multiple sound sources. In the following sections these results are derived using the Fresnel approach along with

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[2]A derivation of how flat the individual radiated wavefronts must be is given in Section 3.3.

further results concerning the prediction of line source array behavior.

## 2 FRESNEL APPROACH FOR A CONTINUOUS LINE SOURCE

The fact that light is a wave implies interference phenomena when an isophasic and extended light source is looked at from a given observation point. These interference patterns are not easy to predict, but in 1819 Fresnel described a way to visualize these patterns semiquantitatively. Fresnel's approach was to partition the main light source into fictitious zones made up of elementary light sources. The zones are classified according to their arrival time differences to the observer in such a way that the first zone appears in phase to the observer (within a fraction of the wavelength $\lambda$). The next zone consists of elementary sources that are in phase at the observer position, but are collectively in phase opposition with respect to the first zone, and so on. A more precise analysis shows that the fraction of wavelength is $\lambda/2$ for a two-dimensional source and $\lambda/2.7$ for a one-dimensional source. (See Appendix 2 for further details.)

In the following, Fresnel's concepts are applied in analyzing the sound field of extended sources. In order to characterize the wavefield for a flat, continuous, isophasic line source at a given observation point, spheres are drawn centered on the listener position with radii that are incremented by steps of $\lambda/2$ (see Figs. 3 and 4). The first radius equals the tangential distance that separates the line source and the observation point. With reference to Figs. 3 and 4, two cases can be observed.

1) A dominant zone appears. The outer zones are alternatively in phase and out of phase. Their size is approximately equal and they cancel each other. Therefore the



**Front view**            **Side view**

Fig. 3. Observer facing line source. Circles are drawn centered on observer O, with radii increasing by steps of $\lambda/2$ (side view). Fresnel zones are defined by the pattern of intersections on source AB (front view).

contribution due to the outer zones can be neglected, and it is assumed that the largest, dominant zone is representative of the SPL radiated by the line source. This is illustrated in Fig. 3, where it is seen that for an observer facing the line source the sound intensity corresponds roughly to the sound radiated by the first zone.

2) No dominant zone appears in the pattern, and almost no sound is radiated to the observer position. Fig. 4 illustrates the case for an off-axis observer.

Moving the observation point to different locations around the line source and repeating the exercise provides a good qualitative picture of the sound field radiated by the line source at a given frequency. It is clear that when there is a circle centered on the listener and locally tangent to the array, the first zone provides the most important contribution to the radiated sound.

Note that the Fresnel representations of Figs. 3 and 4 are at a single frequency. The effects of changing fre-

quency and the on-axis listener position are illustrated in Fig. 5. As the frequency is decreased, the size of the Fresnel zone grows so that a larger portion of the line source is located within the first dominant zone. Conversely, as the frequency increases, a reduced portion of the line source is located inside the first dominant zone. If the frequency is held constant and the listener position is closer to the array, less of the line source is located within the first dominant zone due to the increased curvature. As the observation point is moved further away, the entire line source falls within the first dominant zone.

## 3 EFFECT OF DISCONTINUITIES ON LINE SOURCE ARRAYS

In the real world a line array results from the vertical assembly of separate loudspeaker enclosures. The radiat-
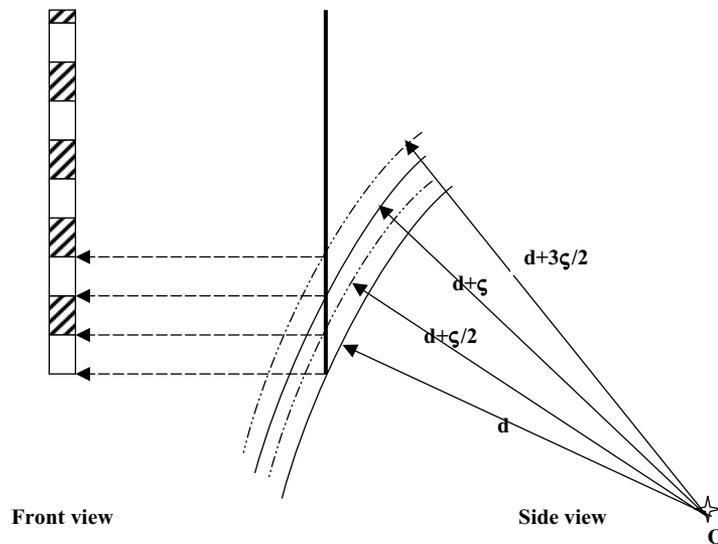


Fig. 4. Observer O is no longer on axis to line source and corresponding Fresnel zones for an off-axis observer O (front view). There is no dominant zone, and individual zones cancel each other.
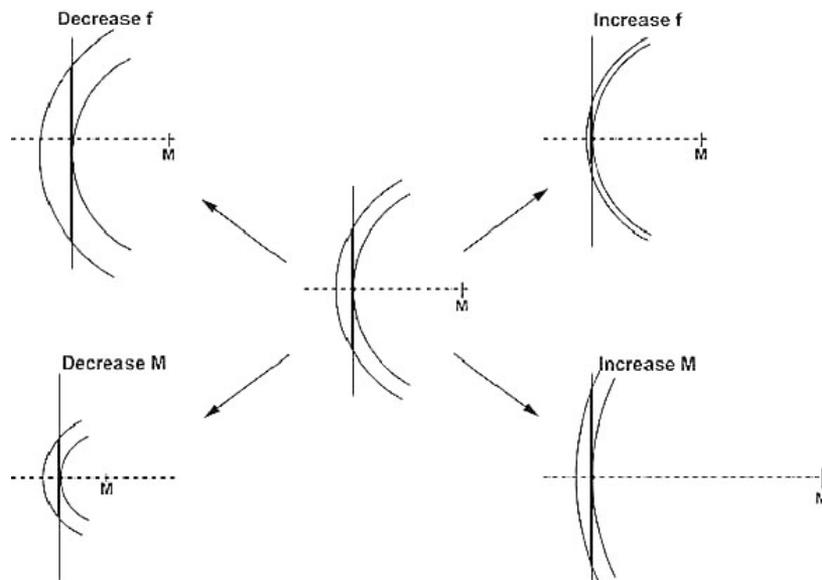


Fig. 5. Effect of changing frequency and listener position.

ing transducers do not touch each other because of the enclosure wall thicknesses. Assuming that each transducer originally radiates a flat wavefront, the line array is no longer continuous. In this section the goal is to analyze the differences versus a continuous line source in order to define acceptable limits for the creation of a line source array.

Consider a collection of flat, isophasic line sources of height $D$, with their acoustic centers separated by the distance STEP. To represent the sound field radiated by this array, the real array is replaced by the coherent sum of two virtual sources, as displayed in Fig. 6. The real array can be considered as equivalent to the sum of a continuous line source and a disruption grid that is in phase opposition with this perfect continuous source.

## 3.1 Angular SPL of the Disruption Grid

The pressure magnitude produced by the disruption grid is proportional to the thickness of the walls of the loudspeaker enclosures. Fig. 7 illustrates how to predict

the effect of the disruption grid in a particular direction at a given frequency. The complex addition of the virtual sound sources of the grid creates an interference pattern that cannot be neglected unless the size of the discontinuities is reduced.

Applying Fresnel analysis from a distant observation point, in this case, the circles crossing the grid become straight lines. Considering the interference pattern as a function of polar angle, for the on-axis direction ($\theta = 0$) all sources appear in phase. At $\theta_{notch}$, half the sources are in phase and the other half are in phase opposition. Thus they cancel each other and the resulting SPL is small. At $\theta_{peak}$ all sources are back in phase and produce an SPL that can be as strong as the on-axis SPL.

Therefore the discontinuities in a line array generate secondary lobes outside the main beamwidth whose effects are proportional to the size of the discontinuities. This is the first reason why it is desirable to attempt to approximate a continuous line source as closely as possible.
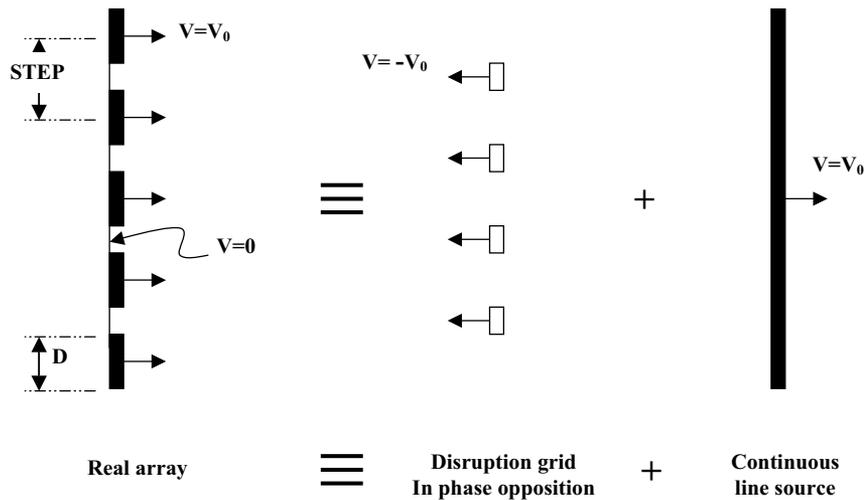


Fig. 6. A real array consisting of sources of size $D$ spaced apart by distance STEP (left) can be considered as equivalent to the sum of a disruption grid and a continuous ideal source (right).
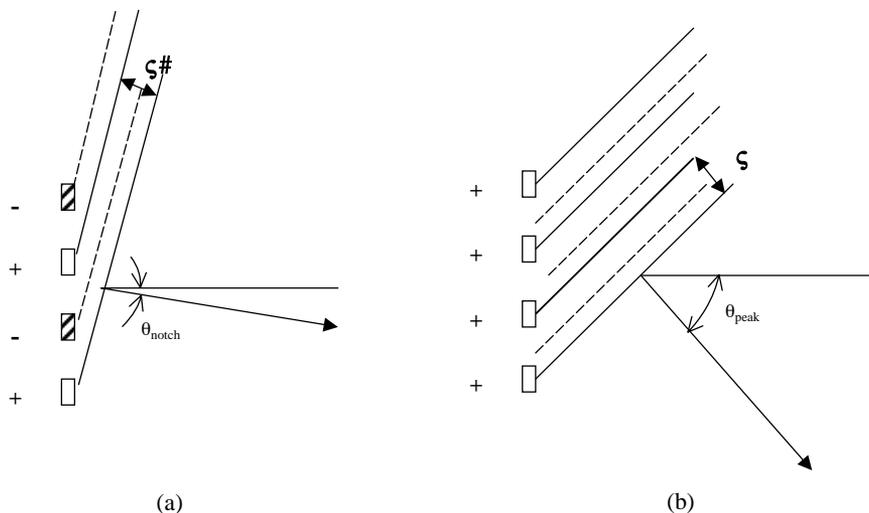


Fig. 7. For a distant observation point, Fresnel circles are transformed into segments. (a) When observing at angle $\theta_{notch}$, half the sources are in phase opposition with the other half, thus producing a null pressure. (b) As we move further off axis, all sources are in phase, thus producing a strong pressure.

From this qualitative approach it is possible to understand that secondary lobes appear in the sound field due to the grid effect. The angles where the secondary peak and the secondary notch arise are given by

$$\text{STEP }\sin\theta_{\text{peak}} = \lambda$$

$$\text{STEP }\sin\theta_{\text{notch}} = \frac{\lambda}{2} \; .$$

In order to avoid a secondary notch in the radiated sound field, it is specified that $\sin\theta_{\text{notch}} \geq 1$. This translates to

$$F \leq \frac{1}{6 \text{ STEP}} \; .$$

As before, $F$ is in kHz and step is in meters. Alternatively, expressing STEP in terms of wavelength,

$$\text{STEP} \leq \frac{\lambda}{2} \; .$$

In other words, the maximum separation or STEP between individual sound sources must be less than $\lambda/2$ at the highest frequency of the operating bandwidth in order for the individual sound sources to properly couple without introducing strong off-axis lobes.

As an example, if STEP = 0.5 m, notches will not appear in the sound field provided that $F < 300$ Hz. In the next section the disruption due to the walls of enclosures is examined and limits are established concerning the spacing between radiating transducers.

## 3.2 Active Radiating Factor (ARF)

The pressure introduced by an ideal continuous line source in the far field is

$$p_{\text{continuous}} \propto H \; \frac{\sin\left(k \dfrac{H}{2}\sin\theta\right)}{k \dfrac{H}{2}\sin\theta} \; .$$

The pressure of the disruption grid is

$$p_{\text{disrupt}} \propto -(\text{STEP} - D)$$

$$\times \; \frac{\sin\left[\left((N+1)k\dfrac{\text{STEP}}{2}\sin\theta\right)\right]}{\sin\left(k\dfrac{\text{STEP}}{2}\sin\theta\right)}$$

where $D$ is the active radiating height of an individual sound source, as shown in Fig. 6.

In the on-axis direction ($\theta = 0$),

$$p_{\text{continuous}}\,(\theta = 0) \propto H$$

$$p_{\text{disrupt}}\,(\theta = 0) \propto -(N+1)(\text{STEP} - D) \; .$$

Since

$$p_{\text{real}} = p_{\text{continuous}} + p_{\text{disrupt}}$$

$$\propto H - (N+1)(\text{STEP} - D)$$

and $H = N\,\text{STEP}$,

$$p_{\text{real}}(\theta = 0) \propto (N+1)D - \text{STEP} \; .$$

At the secondary peak this gives

$$\text{STEP}\sin\theta_{\text{peak}} = \lambda$$

$$k\frac{H}{2}\sin\theta_{\text{peak}} = \frac{2\pi}{\lambda}\frac{N\;\text{STEP}}{2}\frac{\lambda}{\text{STEP}} = N\pi$$

$$p_{\text{continuous}}\left(\theta_{\text{peak}}\right) = H\frac{\sin(N\pi)}{N\pi} = 0$$

$$p_{\text{real}}\left(\theta_{\text{peak}}\right) = p_{\text{disrupt}}\left(\theta_{\text{peak}}\right) = p_{\text{disrupt}}(0)$$

$$= -(N+1)(\text{STEP} - D) \; .$$

It is necessary to define an acceptable ratio for the height of a secondary sidelobe with respect to the main on-axis lobe. A perfect, continuous line source produces secondary sidelobes in the far field that are not higher than $-13.5$ dB compared to the main lobe. This $-13.5$ dB sidelobe rejection ratio is also be specified for the line source array. Therefore it is required that

$$\frac{p_{\text{real}}^2\left(\theta = \theta_{\text{peak}}\right)}{p_{\text{real}}^2(\theta = 0)} \leq \frac{1}{22.4}$$

$$\left[\frac{-(N+1)(\text{STEP} - D)}{(N+1)D - \text{STEP}}\right]^2 \leq \frac{1}{22.4}$$

$$\frac{1 - \dfrac{D}{\text{STEP}}}{\dfrac{D}{\text{STEP}} - \dfrac{1}{N+1}} \leq \frac{1}{4.73} \; .$$

Defining the Active Radiating Factor (ARF) as

$$\text{ARF} = \frac{D}{\text{STEP}}$$

results in

$$\text{ARF} \geq 0.82\left[1 + \frac{1}{4.73(N+1)}\right] \; .$$

Therefore when $N$ is large, ARF must be greater than 82% in order for the secondary sidelobe to be at least 13.5 dB below the main on-axis level. This confirms what was originally obtained in [1]. Note that a secondary sidelobe of only 10 dB below the main on-axis level is obtained

when ARF is equal to 76%. Thus ARF is a factor that must be considered carefully.

When $N$ is large, a practical formula relating ARF to the attenuation of the secondary sidelobes in decibels, Atten(dB), is

$$\mathrm{ARF} = \frac{1}{1 + 10^{-\,\mathrm{Atten(dB)}\,/\,20}}\;.$$

Note the frequency dependence does not show up in the final formulation for ARF since it was assumed that the angle $\theta_{\mathrm{peak}}$ is between 0 and $\pi/2$. However, it should be noted that if the frequency is low enough, there will be no secondary peak, and this is the only way that frequency dependence can enter into the formulation.

### 3.3 The First WST Criteria and Linear Arrays

Assuming that the line array consists of a collection of individual flat isophasic sources, we have just redefined the two criteria required in order to assimilate this assembly into the equivalent of a continuous line source as derived in [1]. These two conditions are termed WST criteria:

- The sum of the individual flat radiating areas is greater than 80% of the array frame (target radiating area), or
- The frequency range of the operating bandwidth is limited to $F < 1/6\mathrm{STEP}$, that is the STEP distance between the acoustic centers of individual sources is less than $\lambda/2$.

Note: Further WST criteria will be derived in the following sections.

For a slot whose width is small compared to its height
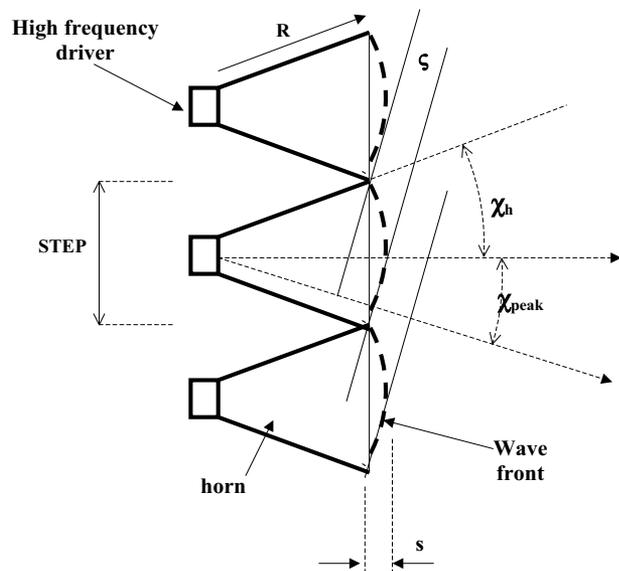


Fig. 8. Vertically arraying conventional horns will not produce a flat wavefront. $\theta_h$ is defined as half the vertical coverage angle for the individual horn, the listener position is at infinity, and the Fresnel circles become straight lines that are tangential to each individual horn wavefront at location $\theta_{\mathrm{peak}}$. This angle $\theta_{\mathrm{peak}}$ corresponds to a peak in SPL when the Fresnel ring spacing is $\lambda$.

$D$, ARF is $D/\mathrm{STEP}$. For the case of touching circular sound sources, the average ARF is $\pi/4 = 75\%$. It is therefore impossible to satisfy the first criterion and for circular pistons the only way to avoid secondary sidelobes is to specify that the maximum operating frequency be less than $1/6D$. In other words, the diameter of a circular piston has to be smaller than $1/6F$. While this is possible for frequencies lower than a few kHz, it becomes impossible at higher frequencies. For example, at 16 kHz we would require adjacent pistons with diameters of a few millimeters.

From this example, it is evident that there is a challenge as to how to fulfill the first criterion at higher frequencies. One solution might consist of arraying rectangular horns so that their edges touch each other however, an important consideration is that such devices do not radiate a flat isophasic wave front. The next question to be answered therefore becomes: how flat does the wavefront have to be in order for the sources to couple correctly?

Considering a collection of vertically arrayed horns that are separated only by their edges, the individual radiated wavefronts exhibit ripples of magnitude $S$, as shown in Fig. 8. The most critical case occurs at high frequencies where the wavelength is small, for example 20 mm at 16 kHz. According to Fresnel, for a far-field observation point the radiated wavefront curvature $S$, should not be greater than half the wavelength, that is 10 mm at 16 kHz.

It was seen in Section 2 that when the circle centered on the listener is also tangent to the wavefront, the SPL is strong. In Section 3.1 it was shown that when the spacing between tangent circles (straight lines in Fig. 8 when the listener position is far away) is $\lambda$, then there is maximum SPL in that direction ($\theta_{\mathrm{peak}}$). If $\theta_h$ is half the vertical opening of the individual horns, there will be no more tangential possibility and therefore no dominant first zone once $\theta_{\mathrm{peak}} > \theta_h$, or

$$\sin\theta_{\mathrm{peak}} > \sin\theta_h$$

As derived in Section 3.2,

$$\sin\theta_{\mathrm{peak}} = \frac{\lambda}{\mathrm{STEP}}$$

and with reference to Fig. 8 we define

$$\sin\theta_h = \frac{\mathrm{STEP}}{2R}$$

where $R$ is the radius of curvature of the individual radiated wavefronts. This gives

$$\frac{\lambda}{\mathrm{STEP}} > \frac{\mathrm{STEP}}{2R} \Rightarrow \frac{\mathrm{STEP}^2}{2R} < \lambda\;.$$

But the sagitta $S$ is

$$S = \frac{\mathrm{STEP}^2}{8R}\;.$$

So we require that

$$S < \frac{\lambda}{4} \, .$$

Fig. 9 displays the calculated SPL versus distance for a line array of 30 horns, 0.15 m high, each producing a curved wave front of 0.3 m (sagitta $S = 10$ mm). Comparison with flat line source shows chaotic behavior of the line array, starting at 8 kHz and increasing with frequency. Apart from severe fluctuations in the SPL at higher frequencies there is also a 4-dB loss at 16 kHz from 10 to 100 m.

Fig.10 illustrates the vertical cross section of the beam width for the same line array in comparison with a continuous line source. Here it is seen that the line array exhibits strong secondary peaks in the near field (20 m) at frequencies higher than 8 kHz.

It is therefore necessary to reduce the departure from a flat wavefront by half ($S < 5$ mm) in order to create an "as good as" perfect line source up to 16 kHz. In effect, this shifts the sidelobe pattern observed in Fig. 10 and
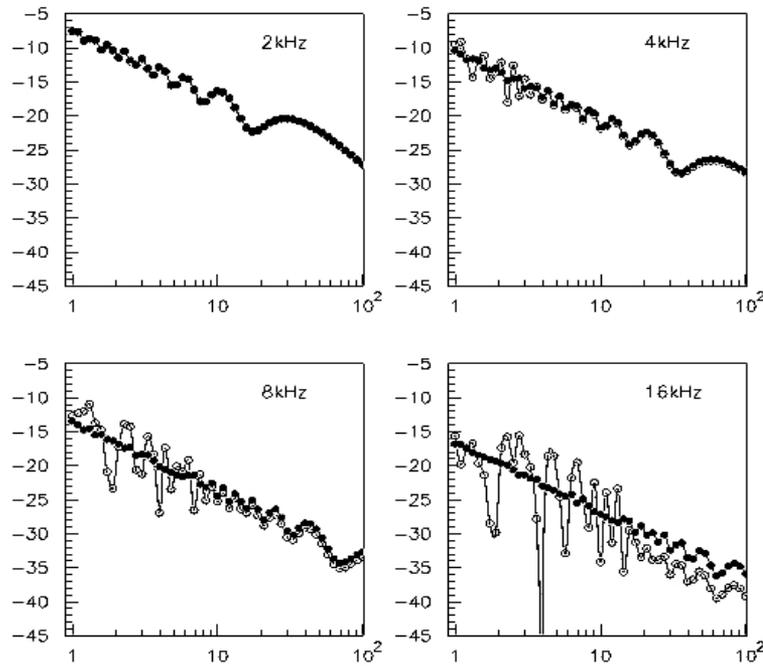


Fig. 9. SPL versus distance for vertical array of 30 horns (total height = 4.5 m, wavefront curvature $S = 10$ mm) calculated at 2, 4, 8, and 16 kHz. ○ Line array; ● continuous line source.
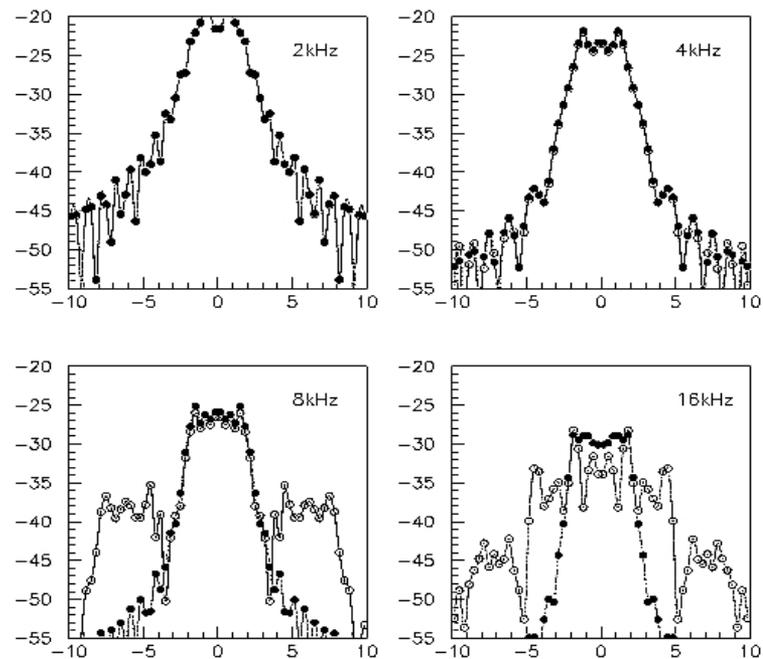


Fig. 10. SPL along a vertical path, 20 m away from vertical array of 30 horns (total height = 4.5 m, wavefront curvature $S = 10$ mm) calculated at 2, 4, 8, and 16 kHz. ○ Line array; ● continuous line source.

the on-axis behavior observed in Fig. 9 from 8 to 16 kHz. We conclude by stating that the deviation from a flat wavefront should be less than $\lambda/4$ at the highest operating frequency (corresponding to 5 mm at 16 kHz).

## 4 SOUND FIELD RADIATED BY A FLAT LINE SOURCE ARRAY

### 4.1 Radiation as a Function of Distance

For the case of a flat line source array we now consider the near-field (cylindrical wave propagation) and far-field (spherical wave propagation) regions and apply Fresnel analysis to derive an expression for the border distance between them. The results obtained can be compared with the analytical calculations presented in [1].

Consider a flat line source array of $N$ discrete elements operating at a given frequency, and an observation point on the main axis of the radiation, as shown in Fig. 11. As the observer moves away from the line source, the number of sources in the dominant zone $N_{eff}$ increases until it reaches the maximum number of available sources ($h = H$). Moving beyond this distance, the number of sources no longer varies.

Each source radiates a sound field as depicted in Fig. 1, and by selecting an observation point in the far field of each source it can be assumed that spherical propagation applies. It will be shown in Section 6.2 that the condition of having the observation point located in the far field of each source implies certain restrictions on the tilt angles between adjacent elements.

The total pressure magnitude $p_{eff}$ is proportional to:

$$p_{eff} \propto \frac{N_{eff}}{d} \text{ ARF STEP}$$

while the SPL is proportional to the square of $p_{eff}$.

In order to express how $N_{eff}$ (or $h$) varies with the listener distance, it can be seen with reference to Fig. 11 that

$$N_{eff} \text{ STEP} = h = \sqrt{4\lambda \left( d + \frac{\lambda}{2} \right)} .$$

For $\lambda \ll d$, two simplified formulations can be derived for the SPL depending on the size of $h$. When $h < H$,

$$I = I_{flat}^{near\ field} \propto \frac{h^2}{d^2} \text{ ARF}^2$$

$$\Rightarrow I_{flat}^{near\ field} \propto \frac{4}{3Fd} \text{ ARF}^2$$

and when $h > H$,

$$I = I_{flat}^{far\ field} \propto \frac{H^2}{d^2} \text{ ARF}^2 .$$

Provided that $N_{eff} < N_{max}$, the SPL decreases as $1/d$, defining the cylindrical wave propagation region. Given this, it is relatively straightforward to extract an expression

for border distance $d_B$,

$$I_{flat}^{near\ field}\left( d_{B\ flat} \right) = I_{flat}^{far\ field}\left( d_{B\ flat} \right)$$

$$\Rightarrow d_{B\ flat} = \frac{3}{4} FH^2$$

where $d_B$ and $H$ are in meters, $F$ is in kHz. The formula derived in [1] for $F \gg 1/3H$, is $(3/2)FH^2$. Therefore Fresnel analysis predicts that the border distance is 50% closer.

When does a near field exist? With Fresnel it can be seen that as the listener distance decreases, the number of sources in the first zone also decreases, except when $\lambda/2 > H/2$, because the entire array is then always in the first zone. Therefore using Fresnel analysis, the same result as found in [1] is derived, that is, there is no near field for $F < 1/3H$.

There is, however, the basic fact that even a continuous source displays ripples in the SPL of the near field, but with magnitude less than $\pm 3$ dB about the average. This is the second reason for assigning ourselves the goal of producing a wavefront as close as possible to a continuous sound source, that is, in order to reduce ripples in the near-field response. Recall that the first reason was to reduce sidelobe levels in the far field.

To illustrate this, consider the line array studied in [3] which consists of 23 dome tweeters with diameters of 25 mm and a STEP distance of 80 mm. The second criterion for arrayability is that the frequency be less than $1/6\text{STEP} = 1/6 \times 0.08 = 2$ kHz. The first criterion is that for frequencies higher than 2 kHz, the ARF should be
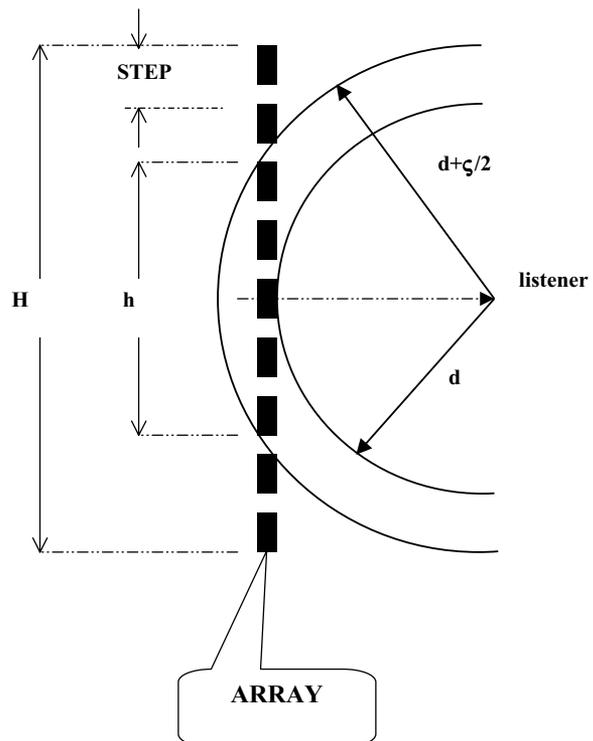


Fig. 11. First Fresnel zone height is $h$. This height grows as distance $d$ increases until $h = H$. At greater distances, no more increase of radiated power is expected.

greater than 80%. Here ARF is too low ($25/80 = 30\%$), and it can be concluded that above 2 kHz this array will exhibit severe problems in the near field.

In Fig. 12 the SPL of a continuous 1.76-m-high line source is compared with the tweeter array as a function of distance at 1 and 8 kHz. It is seen that below 2 kHz the continuous and discrete arrays are similar while at higher frequencies the discrete array shows unacceptable SPL ripples over very short distances. Also shown in Fig. 12 are the $-3$-dB (near field) and the $-6$-dB (far field) lines predicted using Fresnel analysis.

### 4.2 Vertical Pattern in the Far Field

We now investigate the vertical directivity in the far field for a flat line source array. As seen in Figs. 3 and 4, for an off-axis observation point the Fresnel zone pattern on the source can change significantly. This effect is illustrated in the following with some examples.

The pressure in the far field for a flat line source of

height $H$ is expressed as

$$\text{pressure}(\theta) \propto H \frac{\sin\left(\dfrac{kH\sin\theta}{2}\right)}{\dfrac{kH\sin\theta}{2}} .$$

The first notch in pressure is given by

$$\frac{kH\sin\theta}{2} = \pi$$

$$\sin\theta_{\text{notch}} = \frac{2\pi}{kH} = \frac{1}{3FH} .$$

Fresnel analysis allows for the determination of why and where there will be pressure cancellations in the far field. At a given frequency the observation point is rotated around the source at a fixed distance, as shown in Fig. 13.
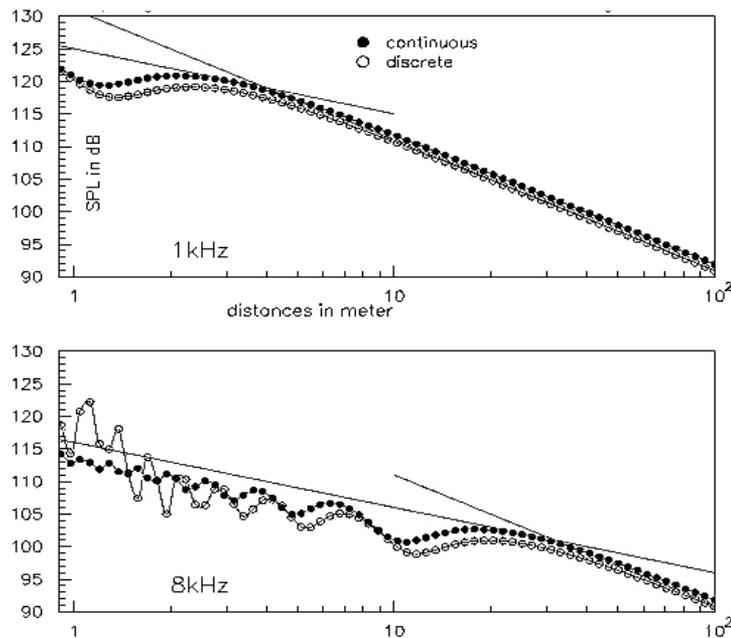


Fig. 12. SPL as a function of distance. ○ 23 tweeters totaling a height of 1.76 m; ● continuous array of same height. The $-3$ dB and $-6$ dB per doubling of distance lines are shown to indicate the border between near and far fields.



(a)                                        (b)
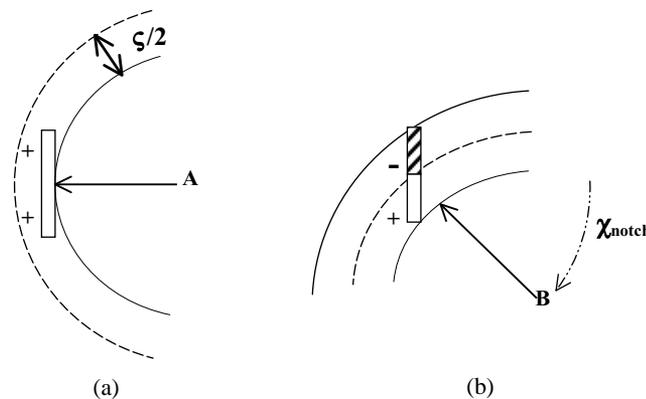
Fig. 13. (a) First Fresnel zone is larger than height of array; entire array is in phase. (b) When rotating around the array there may be an angle where one-half the array is in phase opposition to the other half.

- At position A the observation point is in the far-field region and the entire source is in phase, providing maximum SPL.
- At position B there is cancellation since half the sources are in phase whereas the rest are out of phase. This occurs at the angle $\theta_{notch}$ (Fig. 14).

In order for *IJ* to be equal to $\lambda/2$,

$$IJ = \frac{H}{2}\sin\theta_{notch}$$

$$\sin\theta_{notch} = \frac{\lambda}{H} = \frac{1}{3FH} .$$

This is the same result as obtained in Section 4.2 using the
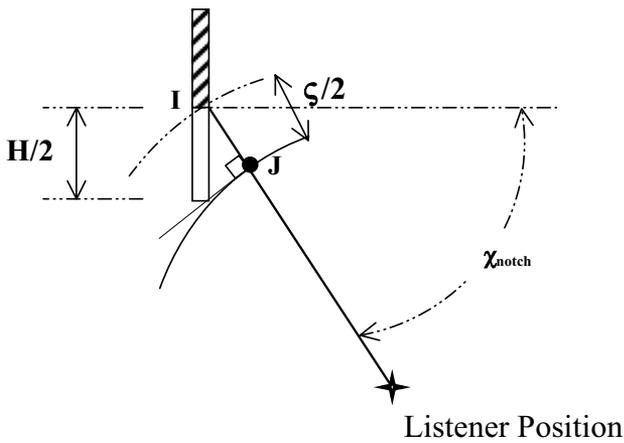


Fig. 14. Defining quantities used to determine $\theta_{notch}$.

analytical formulation.

It should be noted that the Fresnel approach does not give the exact functional behavior of the SPL but provides the characteristic features in a simple, intuitive way. It can be understood physically why there will be an angle where no SPL is produced and the angle can be calculated, but the sin *x*/*x* behavior cannot be derived. For complex situations where more detailed information is desired, numerical analysis techniques must be applied as in [1].

For an on-axis listener position, the sound field is cylindrical up to the border distance $d_B$. Moving the observation point slightly off the main axis can cause the SPL to change significantly. If there are several listeners at different positions and they are aligned on the main axis, then a flat array is acceptable. However, for most applications the audience is more off axis than on axis.

### 4.3 Vertical Pattern in the Near Field

As stated in Section 1 and [1], contrary to the far field, the SPL in the near field is not amenable to closed-form expressions. This is unfortunate since the near field can extend very far, especially at higher frequencies. However, by applying Fresnel analysis, the vertical pattern of the sound field in both the near and far fields can be described.

The SPL in the near field (dotted region ABCD in Fig. 1) will now be considered in greater detail. The SPL as calculated along A′D′ of Fig. 1 is shown in Fig. 15 (black circles). For this example the line source has height $H = 4$ m, distance AA′ is $d = 9$ m, and the frequency is $F = 4$ kHz. Fig. 15 shows that the SPL is nearly constant between A′D′ until it drops to $-6$ dB at the edge of the array. Off axis the SPL decreases by more than 12 dB.
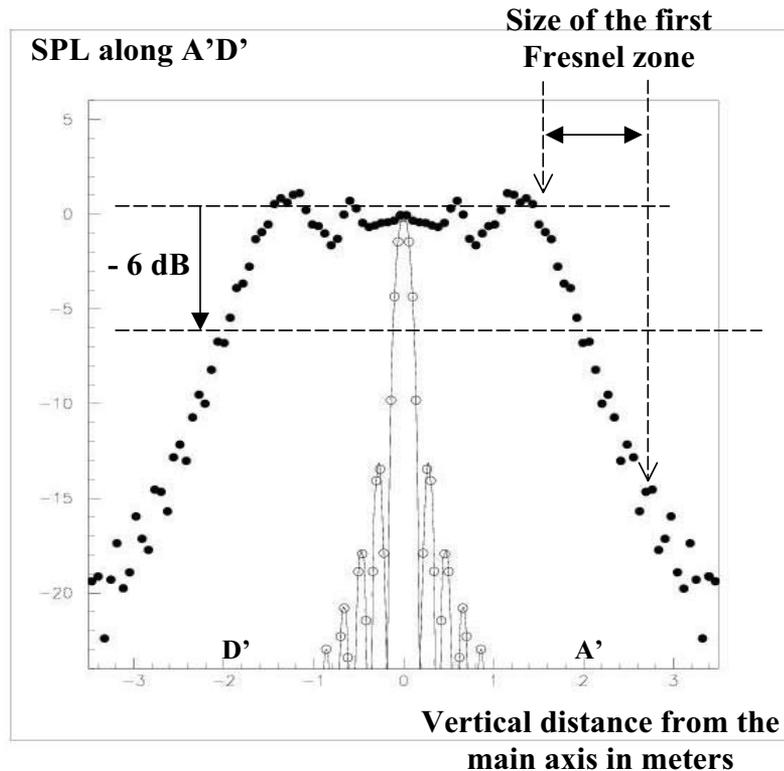


Fig. 15. ● SPL along A′D′ (see Fig. 1) for a line source ($H = 4$ m) at a distance of 9 m, for $F = 4$ kHz; ○ SPL calculated incorrectly using analytical expression for far-field directivity of same line source.

The size of the first Fresnel zone is a very characteristic dimension, and for this example its value is 1.5 m. It is predicted that the SPL will fall to −6 dB at the edge of the array and over half the first Fresnel zone distance. This is seen to be in excellent agreement with the results shown in Fig. 15.

In Fig. 15 the SPL corresponding to the pressure in the far field is also plotted (empty circles),

$$p_{\text{far field}} = \frac{\sin\left(k\frac{H}{2}\sin\theta\right)}{k\frac{H}{2}\sin\theta} \ .$$

This illustrates clearly that a polar plot or an angular formula that is valid in the far field is incorrect in the near field. For further details, see Appendix 1.

## 5 SOUND FIELD RADIATED BY A CURVED LINE SOURCE ARRAY

### 5.1 Radiation as a Function of Distance

Considering a convex line source array of constant curvature, Fresnel analysis can be applied to find the border between the near field and the far field at a given frequency. It will be shown in the following that this border distance depends on the radius of curvature and is always further away for a convex line source than for a flat line source of equivalent length. This somewhat surprising result raises a new question as to how the sound field behaves in the near field with respect to the far field. This question will be answered analytically in the following section, where it will be seen that, in some cases, the transition between near and far field is asymptotic and the difference in the sound field behavior is less pronounced than it is for a flat sound source.

At this point of the discussion it should be noted that an extended sound source and, more specifically, a curved line source array cannot be represented by a point source that radiates a spherical wavefront. Attempts to represent the extended sound source with a point source model necessarily implies compromised results, which turn out to be unacceptable when the sound source becomes large with respect to the wavelength of interest.

With reference to Fig. 16, when the observer is at position A, the flat line source (black vertical line) is not yet entirely in the first Fresnel zone; thus at A the field is cylindrical. Moving to position B, the flat line source is entirely in the first Fresnel zone, and this defines the border between the near and far fields for this kind of source. By comparison, when the array is curved in a convex shape, it can be seen that the border distance is extended since the curved source is not entirely included in the first Fresnel zone.

When the observation point is close to the source, there will be a small difference in the size of the first zone and the flat source will have a slightly larger zone. However, when the listener moves further away, the circles will tend toward straight lines and the difference in the sizes of the first zone will be large and proportional to the inverse of the frequency. Therefore it can be concluded that in the near field the flat and curved sources will display an equivalent SPL whereas in the far field the curved array will have a smaller SPL, which is proportional to the inverse of the frequency.

Thus the far field of a curved array begins farther away than the corresponding one for a flat array. The amount of increase depends on $R$, the radius of curvature of the array. A large value of $R$ implies a border distance slightly further away than for a flat source, as is to be expected, since the flat source is just a particular case of a curved array with an infinitely large radius of curvature. Conversely, for a reduced radius of curvature, the near field can extend very far away from the array. However, the tradeoff is that there is a reduction in the on-axis SPL in comparison with flat line array (see section 6.1).

### 5.2 Vertical Pattern of the Radiated Sound Field

By applying Fresnel analysis it can be seen that a curved line source array projects a uniform sound field,
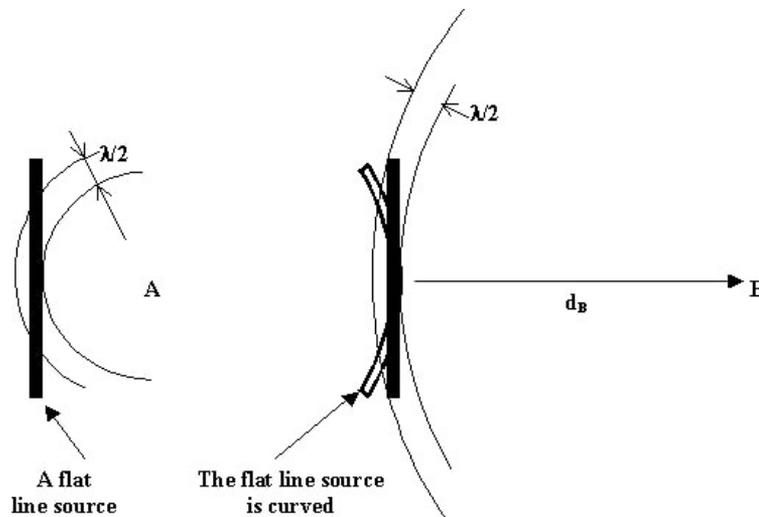


Fig. 16. At position A, wave field of flat line source is cylindrical. At position B, the border between near and far fields has been reached. By curving the array, the border distance is further away than position B since the entire curved line source is not entirely contained within the first Fresnel zone.

except near the edges. Fig. 17 shows an array with constant curvature $R$ and an infinite observation point. Off axis the number of effective sources is the same as on axis until the observation point reaches angle $\theta_{edge}$ and the number drops by a factor of 2. This corresponds to a 6-dB reduction in SPL and therefore defines the vertical directivity of the curved array.

A curved line source array has a uniform SPL that is defined by the angles of its edges whereas a flat line array has a nonuniform SPL but projects a higher level on-axis. Therefore the uniform vertical angular SPL of a curved array has a price, and it is shown in the following that the SPL of a curved line source is less than the on-axis SPL of a flat line source.

Fig.18 shows a comparison between curved and flat line source arrays. The height of both sources is 3 m and the radius of curvature is 5 m for the curved line source. For a frequency of 2 kHz $d_{border}$ for a flat source is situated at 27 m and the SPL is calculated along a vertical line 20 m away from both arrays. The curved source is still producing a cylindrical field at 20 m, and it is seen in Fig. 18 that the vertical pattern of the SPL for the curved source is clearly less chaotic than that of the flat source. However, comparing the average SPLs between $\pm1.5$ m, the flat line array shows a 3-dB advantage over the curved array.



(a)                                                              (b)
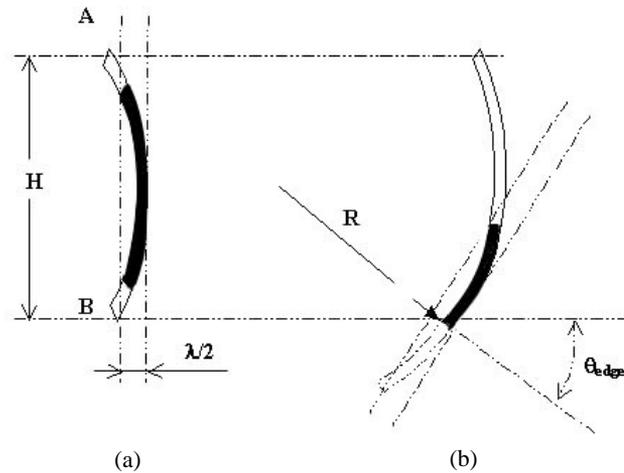
Fig. 17. (a) Curved array AB with the first Fresnel zone shown in black for a distant on-axis observation point. (b) At angle $\theta_{edge}$ the size of the first Fresnel zone is half the size on axis, corresponding to a 6-dB reduction in SPL and defining the vertical directivity of the curved array.
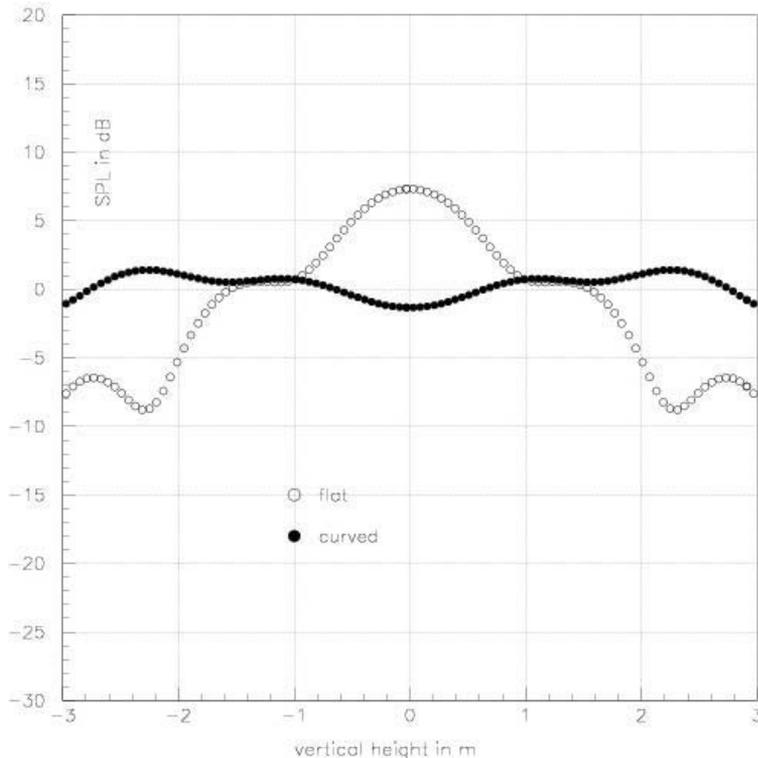


Fig. 18. Comparison of flat and curved line source arrays of the same height. SPL is calculated along a vertical line 20 m from the sources. The curved line source presents less variation in its vertical SPL pattern but with reduced on-axis SPL.

## 6 WAVEFRONT SCULPTURE TECHNOLOGY

Two generic types of extended sound sources have been considered in the preceding sections: the flat line source and the constant-curvature line source. In an effort to adapt the shape of a line source to a specific audience geometry, variable-curvature line sources are now considered. The intention is to focus more energy at the most remote listener positions while distributing the energy better throughout the audience area (see Fig. 19).

### 6.1 Radiation as a Function of Distance

Using Fresnel analysis, the size of the dominant zone is considered at various locations and distances form the array in order to determine how to optimize the shape of the line source to match the audience geometry requirements. In adopting this approach, it is noted that the pressure magnitude of the array at one location is proportional to the size of the dominant zone observed from this position. In previous sections it was seen that the size of the dominant zone is larger for a flat line source and gets smaller as the radius of curvature decreases. To formalize this, the size of the dominant zone is calculated with respect to the number of effective sound sources included in the first zone.

Fig. 20 displays the geometry used to calculate the size of the source, which is included in the Fresnel zone. Recall that the first Fresnel zone is defined by the area of the array contained within the radii $(d, d + \lambda/2)$, where $d$ is the tangential distance from the listener to the array. As in
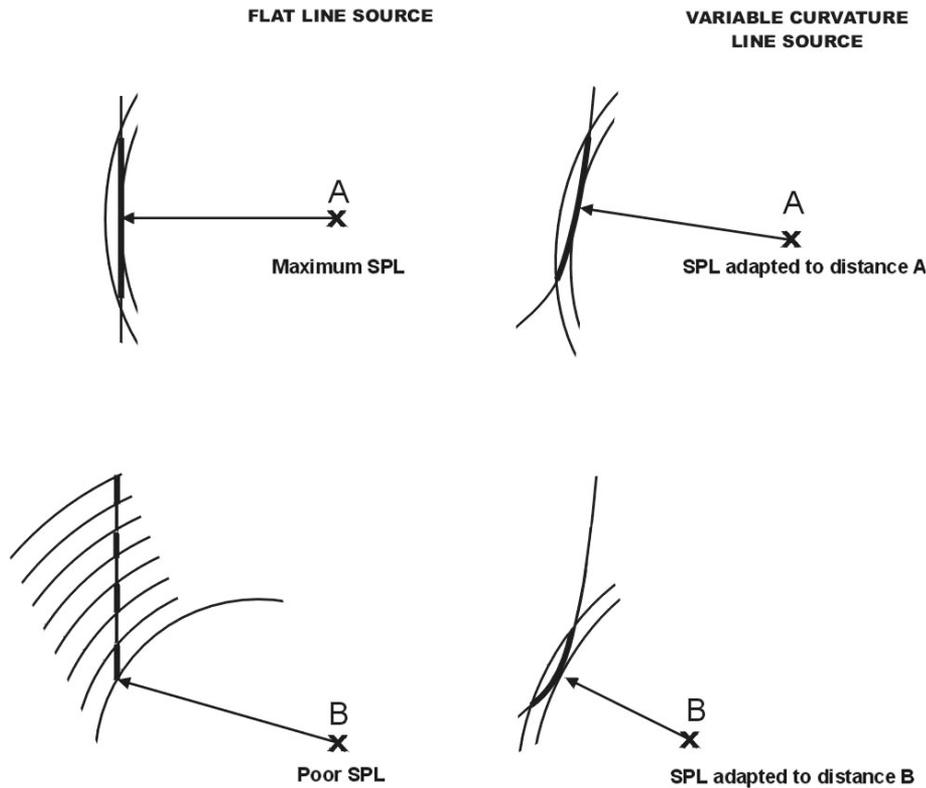


Fig. 19. Comparison of flat and variable-curvature arrays. SPL distribution of the variable-curvature array is adapted to suit the audience geometry.
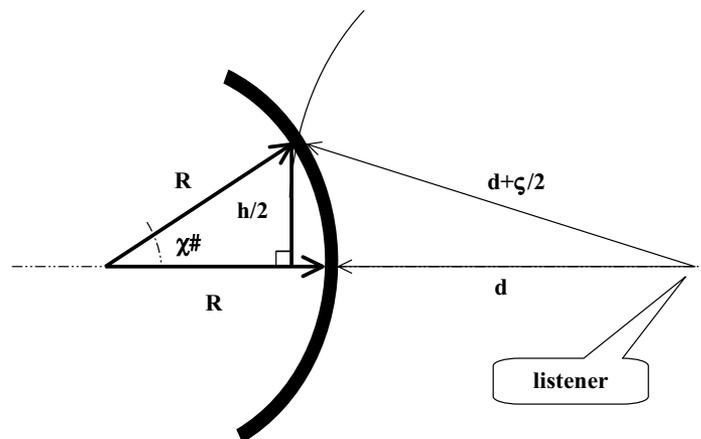


Fig. 20. Nomenclature for calculating the size of the first Fresnel zone for a curved array.

Section 4, the line source array consists of $N$ discrete elements, each radiating a flat isophasic wavefront and operating at a given frequency. These elements are articulated with angular steps to form an array of variable curvature.

As in Section 4.1, the total pressure magnitude is proportional to

$$p_{\text{eff}} \propto \frac{N_{\text{eff}}}{d} \text{ ARF STEP}$$

and the SPL is proportional to the square of $p_{\text{eff}}$.

Referring to Fig. 20, the height of the first Fresnel zone is $h$ and

$$\left(d + \frac{\lambda}{2}\right)^2 = \frac{h^2}{4} + (R + d - R\cos\theta)^2$$

where

$$h = 2R\sin\theta .$$

Using the small-angle approximation for $\theta$ gives

$$\lambda d + \frac{\lambda^2}{4} = R^2 \frac{\theta^4}{4} + R\theta^2 d + R^2\theta^2 .$$

Rearranging the expression,

$$4\lambda d + \lambda^2 = 4R^2\theta^2 \left(\frac{\theta^4}{4} + \frac{d}{R} + 1\right) .$$

The quantity $\theta^2/4$ is smaller than 1 and smaller than $d/R$. Thus,

$$4R^2\theta^2 \approx \frac{4\lambda d + \lambda^2}{1 + \dfrac{d}{R}} .$$

The active height of the first Fresnel zone is

$$h \text{ ARF} \approx 2R\theta \text{ ARF}$$

and the sound intensity from this zone is

$$I \propto \text{ARF}^2 \frac{(2R\theta)^2}{d^2} .$$

We now make the approximation that the closest listener is at a distance $d$, which is larger than the wavelength of interest,

$$I_{\text{curved}} \propto \frac{4\text{ARF}^2}{3} \frac{1}{Fd} \frac{1}{1 + \dfrac{\alpha d}{\text{STEP}}}$$

where STEP is a constant and $d$ is the distance to the listener. Recall that $\alpha$ is the tilt angle between adjacent radiating elements. This angle varies along the array, and the radius of curvature $R$ at a given point on the array is given by STEP/$\alpha$. The expression for a flat line array is obtained

by setting $\alpha = 0$. It is to be noted that this expression is valid provided that the curved line source array is not entirely included within the dominant Fresnel zone. This is the case for

$$\alpha > \frac{4\text{STEP}}{3FH^2} .$$

Conversely, when $\alpha < 4\text{STEP}/3FH^2$,

$$I_{\text{curved}} \propto \frac{H^2 \text{ ARF}^2}{d^2} .$$

This expression for $I_{\text{curved}}$ typically applies at lower frequencies.

Three major results can be derived by comparing the expression for $I_{\text{curved}}$ with the expressions previously derived for a flat line source that is,

$$I_{\text{flat}}^{\text{near field}} \propto \frac{4}{3Fd} \text{ ARF}^2$$

$$I_{\text{flat}}^{\text{far field}} \propto \frac{H^2}{d^2} \text{ ARF}^2 .$$

1) For $\alpha = 0$, the curved array is flat. The two expressions for $I_{\text{flat}}$ and $I_{\text{curved}}$ converge, and both expressions demonstrate near-field behavior with cylindrical sound field propagation.

2) For $\alpha = \text{constant}$, the transition between near field and far field is smooth. At short distances, where $d \ll \text{STEP}/\alpha = R$, the near field is cylindrical and the SPLs of the flat and curved arrays are the same. At greater distances, where $d \gg \text{STEP}/\alpha = R$, the far field typically becomes spherical with an asymptotic limit for $I_{\text{curved}}$, that is,

$$I_{\text{curved}}^{\text{far field}} \propto \frac{4}{3} \frac{\text{ARF}^2 \text{STEP}}{\alpha} \frac{1}{Fd^2} .$$

As predicted by the Fresnel approach in Section 5.1, the far field of a curved array is lower than the equivalent flat array and the decrease goes inversely with the frequency.

3) A constant value for $\alpha d$ can be specified by adapting the angular step $\alpha$ separating two adjacent sound sources to the distance $d$ of their focus target on the audience. See Fig. 21 for an illustration. Setting $\alpha d = K = \text{constant}$ throughout the entire audience profile, the expression for $I_{\text{curved}}$ becomes

$$I_{\text{adapted}} \propto \frac{4}{3Fd} \text{ ARF}^2 \frac{1}{1 + \dfrac{K}{\text{STEP}}} .$$

This expression shows a $1/d$ SPL dependence, thus a 3-dB attenuation per doubling of the distance.

The structure of the sound field has cylindrical effects ($1/d$ dependence) on the audience only, whereas the propagation in a fixed direction (through the air) is in between cylindrical and spherical modes. For this reason the

adapted sound field radiated by a variable-curvature line source having $\alpha d$ constant is termed a pseudocylindrical sound field. Shaping the line source array in such a way so that $\alpha d$ = constant is considered as corresponding to an additional WST criterion, and the method of adapting the sound field to the audience geometry in this manner is termed Wavefront Sculpture.

## 6.2 Limits on the Angular Incrementation of a Curved Line Source

The effect of discontinuities on line source arrays was investigated in Section 3. Another consideration for a variable-curvature line source array is the amount of angular separation that is allowed between two discrete sources before lobing occurs. As shown In Fig. 22, each source individually radiates a near field over a distance that depends on its size and the frequency of interest. The SPL is mainly focused in the dotted regions (recall Fig. 1) and the zone AC is a low-SPL region. This defines a maximum separation angle between two discrete elements, based on the need to project a sound field with no discontinuities on the audience.

Fig. 22 is also of interest since it illustrates that the sound field can be bad above the audience but will be acceptable over the actual audience area.

Note that even if there is no physical gap between the fronts of radiating elements, region AC will still exist due to the fact that the elements are radiating flat wavefronts and are angled with respect to each other.

Let us define $\phi$ as the far-field coverage angle of a single element at frequency $F$. Using a small-angle approximation,

$$\phi = \frac{\lambda}{\text{ARF STEP}} .$$

The distance and angle that separates two adjacent elements are STEP and $\alpha$, respectively. In Fig. 22 the dotted zones represent the sound field of each source and the blank zone AC corresponds to a zone with low SPL. The goal is to reduce the physical extent of the blank zone and to avoid allowing point C to reach the audience.

Using the small-angle approximation, the distance AC
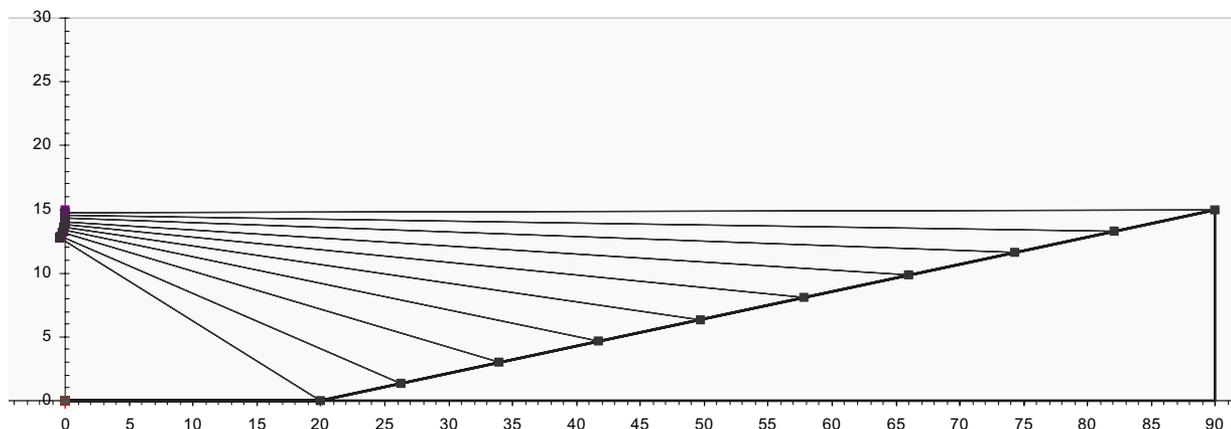
## Elevation (m)



Fig. 21. Wavefront sculpture where a variable-curvature line array is designed so that $\alpha d$ = constant over audience geometry.
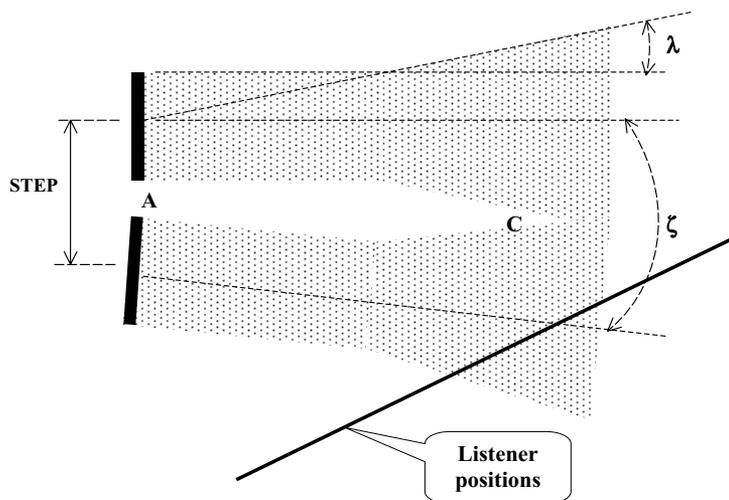


Fig. 22. Two sources separated by distance STEP and tilted by angle $\alpha$ with respect to each other. SPL—dotted region.

is given by

$$AC = \frac{STEP}{2\phi - \alpha} .$$

Rewriting $\phi$ in terms of frequency and specifying that AC is smaller than $d$,

$$\alpha < \frac{2}{3F \ ARF \ STEP} - \frac{STEP}{d} .$$

Where $\alpha$ is in radians, $F$ in kHz, and STEP in meters. It is required that $\alpha$ be greater than zero. Thus,

$$STEP < \sqrt{\frac{2d}{3F \ ARF}} .$$

The worst case is for $F = 16$ kHz and $d = d_{min}$, the minimum distance where a listener will be located. This corresponds to

$$STEP_{max} = \sqrt{\frac{d_{min}}{24 \ ARF}} .$$

Substituting $STEP_{max}$ in the above expression for $\alpha$ we get the following expression for the maximum tilt angle $\alpha_{max}$,

$$\alpha_{max} = \frac{STEP_{max}}{d_{min}} \frac{STEP_{max}}{STEP} \left[ 1 - \left( \frac{STEP}{STEP_{max}} \right)^2 \right]$$

$$= \frac{1}{24ARF} \frac{1}{STEP} \left[ 1 - \left( \frac{STEP}{STEP_{max}} \right)^2 \right] .$$

From this expression it is seen that there is a tradeoff between the maximum element size and the maximum allowable interelement angles, that is, in order to increase the angles between sources, the element size must be reduced.

As an example, consider a minimum listener distance $d_{min}$ equal to 10 m. Since the diameter of a 15 in low-frequency component is typically 0.40 m, this implies a minimum STEP of 0.44 m when allowing for the additional thickness of the loudspeaker enclosure walls. Given this mini-

mum STEP value, the maximum tilt angle $\alpha_{max}$ becomes

$$\alpha_{max} = \frac{5.7°}{ARF} - 2.6° .$$

Since ARF must remain between 0.8 and 1, therefore $\alpha_{max}$ will be between 4.5° and 3.1°, which represents the maximum allowable angle between enclosures. Additional results are tabulated for a variety of component values in Table 1.

The intensity can be expressed as

$$I = \frac{4}{3} \frac{1}{Fd} \frac{ARF^2}{1 + \frac{\alpha d}{STEP}} .$$

Assuming a STEP of 0.44 m, $d_{min}$ of 10 m, and a frequency of 16 kHz, it is found that $\alpha d/STEP$ is of order 1. Therefore the intensity will be roughly a factor of 2 smaller ($-3$ dB) than the on-axis intensity for a flat array ($\alpha = 0$).

## 7 CONCLUSION

The Fresnel approach in optics has been applied to acoustics in order to understand and characterize the sound field radiated by linear arrays. Fresnel analysis does not provide precise numerical results but gives a semi-quantitative, intuitive understanding. More precise results must be obtained using numerical analysis techniques. However, Fresnel analysis allows one to predict the answers in a semiquantitative way.

The problem of defining when an assembly of discrete sources can be considered as equivalent to a continuous line source was addressed, and the reasons why a continuous line source is desirable were also presented. It was seen that a continuous line source exhibits two different regimes: when close to the source the SPL varies as $1/d$ (cylindrical wave propagation) and far away the SPL varies as $1/d^2$ (spherical wave propagation). The position of the border between these two regimes is proportional to the frequency and to the square of the height of the array. In addition, for low enough frequencies there is no near field.

By studying the properties of curved arrays using Fresnel analysis, the conditions concerning the tilt angles

Table 1.

| Component (in) | Nominal Component Diameter (mm) | Nominal Enclosure Height (m) | Maximum Interelement Angle[*] (degrees) | Maximum Interelement Angle[†] (degrees) |
|---|---|---|---|---|
| 18 | 460 | 0.496 | 3.2 | 4.6 |
| 15 | 380 | 0.416 | 4.8 | 6.0 |
| 12 | 300 | 0.336 | 7.0 | 7.9 |
| 8 | 205 | 0.241 | 11.0 | 11.7 |

[*]$d_{min} = 10$ m
[†]$d_{min} = 20$ m.

between loudspeaker enclosures and the size of these enclosures required in order to provide a uniform cylindrical SPL over a given audience geometry was determined.

Let us summarize the WST criteria for arrayability.

- Flat array

    1) Either the sum of the individual flat radiating areas covers more than 80% of the vertical frame of the array, that is, the target radiating area,

or

    2) The spacing between the acoustic centers of individual sound sources is smaller than $1/6F$, that is, less than $\lambda/2$ at the highest operating frequency,

and

    3) The deviation from a flat wavefront should be less than $\lambda/4$ at the highest operating frequency (corresponding to 5 mm at 16 kHz).

- Curved array, the same criteria as for the flat array apply. In addition,

    4) In order to have cylindrical coverage over a given audience geometry, the product of enclosure tilt angle and throw distance ($\alpha d$) should be constant.

    5) The maximum vertical cabinet size is given by

$$\text{STEP} < \sqrt{\frac{2d_{\min}}{3F_{\max}\,\text{ARF}}}\;.$$

For $F_{\max} = 16$ kHz and ARF $= 80\%$ this gives

$$\text{STEP} < \text{STEP}_{\max} = \sqrt{\frac{d_{\min}}{20}}\;.$$

Assuming STEP $\ll$ STEP$_{\max}$, the enclosure tilt angles should not exceed

$$\alpha_{\max} = \frac{3°}{\text{STEP}}\;.$$

Therefore, in order to increase the relative angle between enclosures, it is necessary to reduce the vertical size of the enclosures.

It is not possible to have a well-behaved SPL everywhere. However, as demonstrated in this paper, it is still possible to physically curve an array in accordance with the preceding criteria so that a homogeneous and cylindrical SPL is delivered over a given audience area. The five defined criteria are termed Wavefront Sculpture Technology and the approach of physically shaping the array to adapt coverage and SPL distribution is termed Wavefront Sculpture.

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[5] L. L. Beranek, ACOUSTICS, 3rd printing (American Institute of Physics, New York, 1990).

## APPENDIX 1

It is only in the far-field region that directivity, polar plots, and secondary lobes can be defined. In the near field these concepts can be greatly misleading due to the fact that the line source cannot be represented as a point source and a polar diagram makes the assumption that the energy flow is radial. For example, in order to draw polar plot, it is necessary to measure the SPL along the arc of a circle as pictured in Fig. 23 (b). This would result in the polar plot as shown in Fig. 23 (a), and it would be wrongly concluded that a large fraction of energy is being radiated



(a)                    (b)
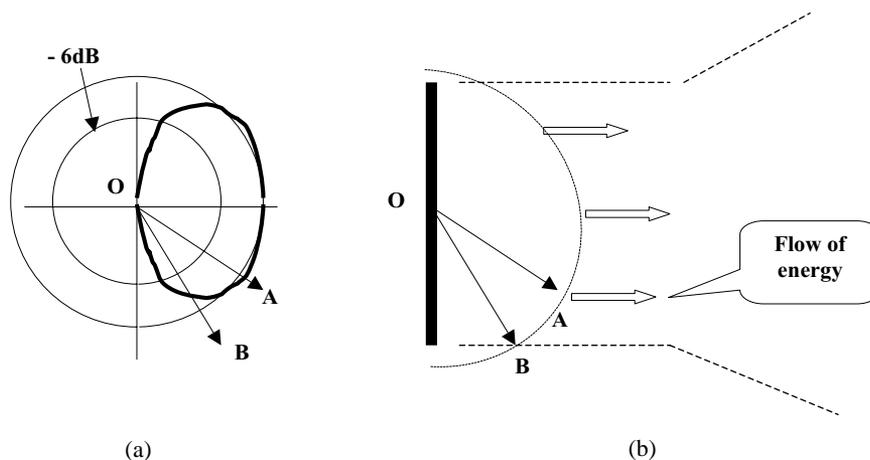
Fig. 23. (a) Polar diagram where flow of energy is supposed to emanate from point source O (along OA, for instance). (b) Using such a polar diagram in the near field would be incorrect for characterizing a line source.

above and below the line array. This is incorrect since in the near field the energy flow is in the forward direction, perpendicular to the line array.

## APPENDIX 2

To conduct Fresnel analysis, circles with $\lambda/2$ increments in their radii are drawn, centered on the observation point. This may appear somewhat surprising since half a wavelength leads to a phase opposition. One edge of the zone is in phase opposition to the other edge of the zone, and consequently a small resultant SPL would be expected. Qualitatively, it can be demonstrated why Fresnel chose that value and why the SPL is not small but, on the contrary, reaches it's maximum level.

Consider the first zone as divided into small but finite pieces, as shown in Fig. 24. OA is the resultant SPL from the first Fresnel zone and is larger than OB, which incorporates part of the second zone.

In order to see this more rigorously, the SPL is calcu-

lated for a continuous line source with variable height. The observation point is at a distance of 4 m on axis (see Fig. 25).

The SPL due to $H(\Delta r)$ is normalized by the SPL arising due to the first Fresnel zone at the same distance, and it is assumed that the entire zone is at the center. This is equivalent to neglecting the $\lambda/2$ variation from the center to the edge of the zone. Results are pictured in Fig. 26 where it is seen that the maximum SPL is not reached for $\lambda/2$ but for $\lambda/2.7$. This difference is due to the fact that Fresnel considered two-dimensional sources whereas we consider only one-dimensional sources. Since we are only interested in the qualitative predictions of the method, $\lambda/2$ is maintained as the reference value for convenience. This is considered a reasonable approximation since Fig. 26 shows that the SPL difference between $\lambda/2$ and $\lambda/2.7$ is only 0.5 dB.

Finally, it should be noted that, on average, the light intensity from the dominant zone is roughly 6 dB higher than the light intensity from the complete source.
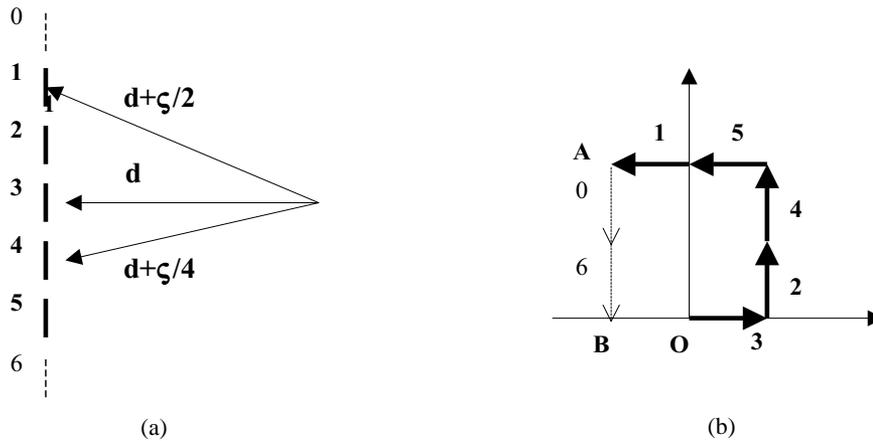


Fig. 24. (a) First Fresnel zone broken into 5 segments of equal sound pressure. (b) Argand diagram of complex amplitudes associated with these segments and their resultant sum.
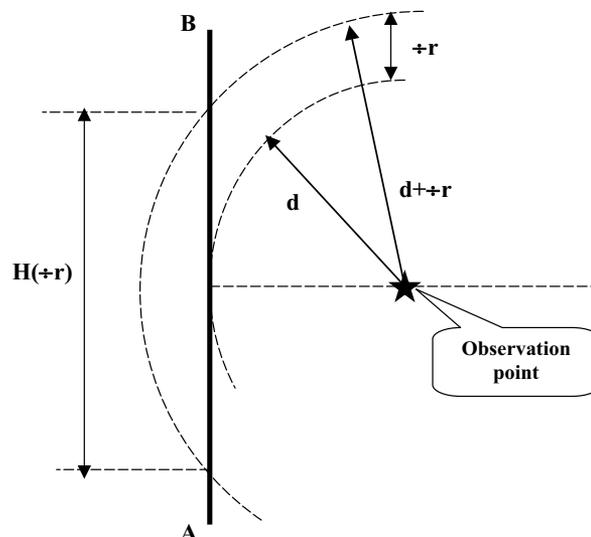


Fig. 25. Circle of radius $d$ drawn from the observation point, tangent to line source AB. Drawing a circle of radius $(d + \Delta r)$ defines a segment of height $H$ on AB.

**SPL(H(÷r))/SPL first zone**
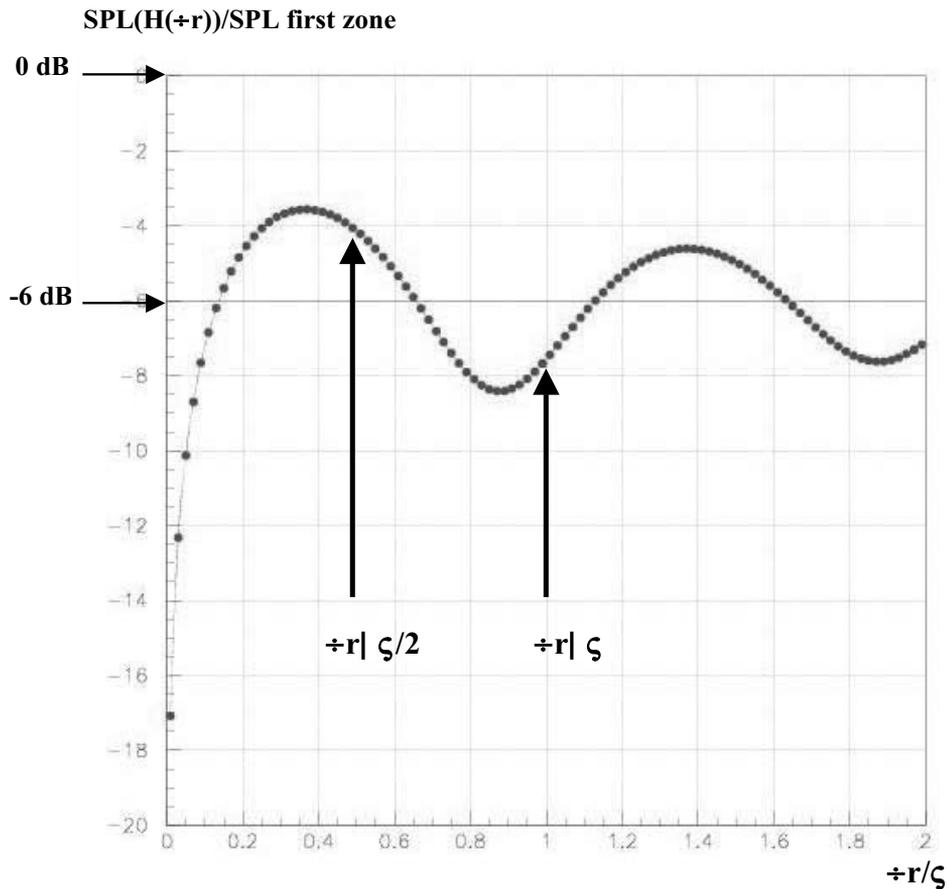


Fig. 26. Normalized SPL of segment $H(\Delta r)$, defined in Fig. 25, displayed as a function of increase in radius of circle.

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**THE AUTHORS**



M. Urban
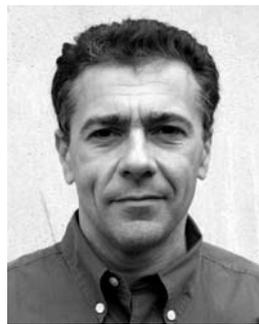
C. Heil

P. Bauman

Marcel Urban studied at the University of Orsay, Paris, France, from which he received a Ph.D. degree in experimental physics in 1973 and a Ph.D. degree in theoretical physics in 1977. He is presently director of research at the Centre National de la Recherche Scientifique (CNRS) in France. His principal area of interest is the study of high-energy cosmic rays and he conducts research in conjunction with the AUGER observatory in Argentina. Dr. Urban has been a consultant for L-ACOUSTICS since 1983.

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Christian Heil graduated from the University of Science of Orsay, Paris, France. He obtained a Ph.D. degree in physics in 1981, while in the department of Elementary Particles, under the direction of Dr. Urban.

He then joined Thomson Laboratories for a three-year research program on plasma physics. He also became increasingly involved in professional audio consulting, working for various corporations in the area of loudspeaker design. In 1984, Dr. Heil founded L-ACOUSTICS, where he is currently both chairman and director of R&D.

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Paul Bauman obtained a master's degree in physics from the University of Waterloo, Ontario, Canada, in 1985 and a master's degree in electrical engineering from the McMaster University in 1991. Over the past 20 years his professional audio activities include loudspeaker design, measurement and manufacturing, sound reinforcement system design, live sound reinforcement system engineering

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Mr. Bauman was the AES Toronto Section secretary (1986 to 1990); secretary for the AES 7th International Conference, Toronto, Canada, in 1989; and workshop chairman for the AES 108th Convention, Paris, France, in 2000; and he is currently a member of the AES Technical Committee on Acoustics and Sound Reinforcement.