Calculating the Wavelength Response of the Magnetic Recording and Reproducing Processes – a Historical Review

[http://www.aes.org/aeshc/pdf/mcknight_calculating-wavelength-response.pdf]

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Measurement shows that the unequalized flux-*vs*-wavelength response of a magnetic tape first falls at 6 dB/octave, then at 12 dB/octave with decreasing wavelength. Early researchers thought that this was due to the reproducing gap loss, but practical reproducers use short enough gaps that the gap loss is negligible. In 1951, Wallace hypothesized that the tape magnetization was constant with depth into the coating, and he calculated the resulting "Wallace thickness loss" that occurs, but this gives only a 6 dB/octave decrease. In 1959, Tjaden hypothesized that the magnetization is actually 0.2 at the tape surface, and increases with depth into the coating. This results in a "Tjaden thickness loss" that corresponds to the measured response. Finally, in 1974...1978, Bertram rediscovered and expanded Tjaden's theory, and also discovered the effect in reproduction of the anisotropic reversible permeability (different in the directions "down the length" and "thru the depth" of the medium). I have programmed Bertram's equations – the "Bertram thickness loss" – for calculating the short-wavelength response.

0 INTRODUCTION

A very obvious and important characteristic of any audio system is its frequency response. With a recording system using a moving medium, frequency is converted into a recorded wavelength, so "frequency response" becomes "wavelength response".

The best-known high-frequency (actually, short-wavelength) response factor of a tape reproducing system is the "gap loss" of the reproducing head, and some of the early magnetic recording papers say that the gap loss is the major short wavelength loss. I have discussed this previously [1], and shown that, in fact, with most practical reproducers, the gap loss is but a very small (and easily correctable) factor, so I will not further considered it here.

In an earlier paper [2] written when I was at Ampex Corp, I defined "frequency response" and "wavelength response" of a magnetic tape audio recorder/reproducer with a moving medium, where wavelength, λ, is the recording speed s divided by the recording frequency f. Fig. 1 shows a typical measured unequalized wavelength flux response[1] for a "mastering" tape (from Fig. 6 of [3]). Note that the response falls with decreasing wavelength at a rate of 6 dB/octave at first, then, two octaves later, at 12 dB/octave.

The wavelength flux response (upper scale) does not depend on the tape speed, whereas the frequency response without equalization (lower scale) is very much a function of the tape speed.

I then reviewed the frequency response of magnetic recorders for audio – how you standardize the reproducing system (the reproducing head, amplifier, and equalizer), and the recording system (recording amplifier, equalizer, head, and tape) .

Then I showed how the equalizer frequency responses are determined: the reproducing equalizer is basically an "integrator" – that is, a response falling 6 dB/octave, to compensate for the 6 dB/octave rise from the "differentiation" of Faraday's law in the inductive reproducing head. Therefore the response of this system is flux proportional. This is then modified at the upper frequencies (above a few kilohertz) usually by a 6 dB/octave high-frequency lift, with the "transition frequency" (+ 3 dB frequency) being one of the standard equalizations defined by the International Electotechnical Commission (IEC) [4], which includes the earlier standards of the (US) National Association of Broadcasters (NAB), and the International Radio Consultative Committee, known by its French initials CCIR.

Then the recording equalization is the difference between the frequency response of the unequalized system, and the standard reproducing equalization.

Finally I discussed the theory of the "Wallace thickness loss" response, and noted that when you compare the then-current (1954...1960) theory for the wavelength response of the tape with the measured wavelength response, you find a large difference, but no-one that I knew of at the time could explain the difference.

Since then, the source of the difference has been found, and a new theory (the "Tjaden thickness loss") that agrees with measurements was published in two

[1] Some papers – for instance the 3M "SoundTalk Bulletin Nr 1" http://www.aes.org/aeshc/docs/3mtape/soundtalk/soundtalkbull01.pdf – instead show the output voltage of the reproducing head, and show a curve rising 6 dB/octave, flattening out, then falling 6 dB/octave. The data are the same – only the presentation is different.

papers that I was then not aware of in 1959 and 1963, plus several papers in 1974...1978 (the "Bertram thickness loss"). I will review the old theory, and present the new theory, which includes experimental verifications.

1 THE ORIGINAL THEORY: THE LONGITUDINAL RECORDING FIELD GIVES CONSTANT MAGNETIZATION VS DEPTH

Wallace [5], at Bell Telephone Labs in 1951, first set out to calculate and measure the "spacing loss", which is the effect in reproduction of a physical space between the magnetically recorded medium and the reproducing head face.

Then he calculated the response in recording and reproducing by assuming that the medium is uniformly magnetized throughout its thickness, and envisioning a recorded tape as consisting of many hypothetical thin layers, and applied the spacing loss to each layer, summing the flux from all of the layers to get a total short-wavelength response. He called this theory the "thickness loss", because it depends on the thickness of the magnetic coating. The "Wallace thickness loss" response function [W(*kd*)] is:

$$W(kd) = (1 - e^{-kd})/kd, \qquad (1)$$

where

$k = 2\,\pi f/s$
f = recording frequency
s = tape speed in recording
d = depth of recording = thickness of the tape coating
e ~ 2.72 ("Euler's number")

Thus *kd* is the frequency normalized to the tape speed and the coating thickness.

He compared this recording theory with a response that he measured, and found that *by arbitrarily assuming a certain spacing between head face and the coating*, the two could be made to agree pretty well.

He did comment that "very little is known about the magnetization pattern in the medium". For his theoretical calculations, Wallace "implicitly assumed...that the medium is uniformly magnetized throughout its thickness, and this may not be the case".

2 THEORETICAL RESPONSE DOESN'T MATCH MEASUREMENTS

Most researchers and writers of books have adopted Wallace's assumption that the medium is uniformly magnetized throughout its thickness, and most magnetic recording texts at this time (2012) still give the reproducing gap loss and the "Wallace thickness loss" as the cause for the short-wavelength response in tape recording and reproducing.

I noticed in 1960 [2], however, that the calculated response according to Wallace's theory deviates considerably (9 dB) from the measured response at a 10 μm (375 μinch) wavelength, which is 20 kHz at 7.5 in/s. This is shown in Fig. 2. But at the time I was not aware of any research that provided a correct response, or an explanation of the observed difference.

As further evidence of a problem with Wallace's theory, Daniel and Levine [6], in 1960, compared a calculated theoretical and a measured maximum long-wavelength recording sensitivity for 13 different tapes, and found a fairly consistent ratio of calculated sensitivity to measured sensitivity the about 2:1. Again, they were not able to find a reason for this gross difference.

It seemed curious at the time that no-one seemed to have noticed that the "Wallace thickness loss" curve falls at 6 dB/octave, whereas the measured wavelength response falls asymptotically to 12 dB/octave, as shown above in Fig. 1.

3 THE NEW THEORY: THE TOTAL RECORDING FIELD GIVES MAGNETIZATION THAT INCREASES LINEARLY WITH DEPTH

3.1 Early research by Tjaden

Unbeknownst to us at Ampex at the time, someone *had* noticed the inconsistency in wavelength responses: D. L. A. Tjaden, of Philips Research Laboratories, in a little-known 1959 paper from a meeting that is rather obscure for magnetic recording research [7], hypothesized that the recording occurs in a semi-circular zone around the recording gap where the bias field equals the coercive force of the tape, and that the *total* bias field – both the vertical and longitudinal recording components – is important, not just the longitudinal bias field as assumed by Wallace. This, surprisingly, gives a recorded longitudinal magnetization of *nothin*g at the tape surface, increasing linearly with the depth into the coating. The vertical component of the magnetization is 0.2 at the tape surface[2], falling to nothing at the back of the tape. The total magnetization, M, is the vector sum of the two components, as shown in Fig. 3.

Under this condition, the "Tjaden thickness loss" [T(kd)] wavelength response would be:

$$T(kd) = [1 - (1 + kd)\, e^{-kd}] / (kd)^2 , \tag{2}$$

as shown in Tjaden's Fig. 3 from [7], and curve a) of Bertram's Fig. 2 from [11], reproduced below in Fig. 4, where the response at first falls at 6 dB/octave, then later falls asymptotically to 12 dB/octave, which is consistent with measurements.

In a well-known paper in 1964 [8], Tjaden and Leyten, built a 5000:1 scale model of the magnetic recording process, to test the hypotheses of the earlier paper. Their measurement (Fig. 14 from [8]) verifies the earlier paper's hypothesis [7] about the magnetization – that it is 0.2 at the tape surface, rising linearly with the depth into the tape, as is shown in Fig. 3.

In Tjaden's later paper, he does not discuss the effect that this would have on the wavelength response, because he had previously done this in his earlier paper [7].

3.2 Later research by Bertram

In 1974, Neal Bertram, then at Ampex, independently conceived this idea that the total bias field determines the recording point, and published his paper "Long Wavelength AC Bias Recording Theory" [9].

In this paper, Bertram first uses Wallace's assumption of constant magnetization at all depths (see Fig. 3, below, the top line "longitudinal model"). He calculates the recorded flux vs bias field for various ratios of the coating thickness to the recording gap length, and notes that the calculated values differ very considerably from measured data. He further notes that the recording sensitivity calculated from the longitudinal recording theory is twice the value measured by Daniel and Levine [10].

[2] This is because most tapes are now longitudinally oriented, mechanically by the coating process, and magnetically with an orienting magnet while the binder is drying. This orientation increases the longitudinal susceptibility (the recording sensitivity), and decreases the vertical susceptibility.

He then hypothesizes a "total field" recording model, and repeats the calculations. These show, as Tjaden had found in his earlier papers, that the recorded longitudinal magnetization at the tape surface is zero (no magnetization), and the total magnetization increases linearly to a maximum at the back of the coating, as shown in the lower curves of Fig. 3.

In a second paper, "Wavelength Response in AC Biased Recording" [11], Bertram calculates the long-wavelength sensitivity and the wavelength response using the "total field" model, and compares them with the "longitudinal field" model. He shows in his figure reproduced below as Fig. 4 that the total field model gives a calculated sensitivity and response – the "Tjaden thickness loss" – that agree well with measured data (the circles and dots on the graph).

Bertram's parameter "reduced wave number (kd)" is proportional to frequency: $k = 2\pi/\lambda$, where $\lambda = s/f$ is the recorded wavelength, and d is the recording depth, which equals the tape coating thickness when the tape is biased for maximum sensitivity at long-wavelength. Reduced wave number $kd = 10$ corresponds to 15 kHz for a cassette system (speed s = 48 mm/s, and coating thickness $d = 5$ µm).

Bertram submitted this paper to a magnetics conference, probably in 1975, and received back a polite note from Tjaden (who was also presenting a paper at that conference), telling him of his 1959 paper [7] that reports essentially the same conclusions. This is when we first learned of Tjaden's earlier work.

4 THE EFFECT IN REPRODUCTION OF THE ANISOTROPIC REVERSIBLE PERMEABILITY OF THE MEDIUM

Then, in 1978, Bertram added the final detail, the effect in reproduction of the anisotropy factor and the mean permeability of the tape [12]. "Anisotropy" of the permeability simply means that the permeability in the direction of the tape travel μ_x , is different from the permeability into the depth of the tape, μ_y .

Interestingly, all of the effects up to this point have *decreased* the recorded short-wavelength flux; but this effect *increases* the reproduced short-wavelength flux.

Decreasing value of α and increasing the value of β both increase the short-wavelength response, as shown in Fig. 5, below.

Fig. 4 assumed permeabilities of 1 (the lowest curve in Fig. 5). The second curve up in Fig. 5 assumes a mean permeability $\beta = 2.25$, which is typical of modern tapes. The next four curves assume decreasing values of α (corresponding to increasing ratios of μ_y / μ_x), which produce increasing flux at the shorter wavelengths. (The response calculation actually uses $\alpha = \sqrt{(\mu_x / \mu_y)}$).

The "Bertram thickness loss" response function B(kd) for linearly increasing magnetization with depth, an anisotropic medium, and a head-to-tape spacing of "a" (from [12] equation 22) now becomes:

see **(3)** on next-to-last page, to be placed **here**

where:

s = tape speed

f = recorded frequency at this speed

d = the recording depth (= coating thickness when biased for maximum long wavelength sensitivity)

μ_x = reversible relative permeability in the head-to-medium motion direction μ_y = reversible relative permeability in the direction vertical to the medium plane $\alpha = \sqrt{(\mu_x / \mu_y)}$ = anisotropy factor $\beta = \sqrt{(\mu_x \mu_y)}$ = mean permeability

a = head-to-tape spacing.

Neal Bertram has provided the data in Table 1, below, of measured values of permeability for 9 commercial types of tape available at the time (the 1990s).

We don't have the permeabilities for all kinds of tapes, and this measurement requires special equipment. But if the value permeability is not given in Table 1, one could use an approximate value of $\alpha = 0.8$ and $\beta = 2.25$.

5 CALCULATING THE TAPE RECORDING AND REPRODUCING FREQUENCY RESPONSE

In order to calculate the tape recording and reproducing response from this formula, we need the values for the tape coating thickness, the permeabilities, the head-to-tape spacing, and a calculating program.

The Table 1 of permeabilities is below. I have somewhat arbitrarily taken a head-to-tape spacing of 10 nm. I have also added some results of the calculations for the frequencies 2.5 kHz, 5 kHz, and 10 kHz, at a tape speed of 190 mm/s (7.5 in/s).

I have programmed this calculation of the "Bertram thickness loss" in Forth for DOS [13]. (There are now free programs – for instance "vDos" – that will let you run DOS programs on Windows 10.) Since practical results should be in terms of frequency, not wavelength, I have provided this in the program. This program would be more practically useful if it included the effects of the pre- and post-emphasis equalizations – I hope to add those some day. The frequency response of the "thickness loss" is a function of (in consistent units):

Table 1: Reversible Permeability Values, Thickness, and Calculated Relative Flux Level at Three Wavelingths. In order of increasing coating thickness

Tape Type	μ_x	μ_y	$\mu_y/\mu_x = 1/\alpha^2$	$\alpha = \surd(\mu_x / \mu_y)$	$\beta = \surd(\mu_x \mu_y)$	Coating Thickness/ [µm]	Relative Flux Level/[dB] for: Recorded wavelength/[µm] 76 = Reproduced Frequency @ 190 mm/s /[kHz] 2.5	38 / 5	19 / 10
Ampex 705	1.77	3.26	1.83	0.74	2.40	4.3	-0.8	-2.0	-4.7
Ampex 642	1.72	2.68	2.30	0.66	2.15	10.7	-2.3	-5.4	-11.5
3M 111	3.02	3.49	1.16	0.93	3.25	11.2	-3.3	-8.0	-16.4
Ampex 406	1.75	1.99	1.13	0.94	1.87	12.7	-4.6	-9.8	-18.6
3M 206	2.01	2.42	1.21	0.91	2.21	13.7	-4.6	-10.1	-19.2
Ampex 456	1.56	3.67	2.37	0.65	2.39	14.0	-3.1	-7.1	-14.6
BASF 911	1.93	3.18	1.65	0.78	2.48	16.0	-4.5	-10.0	-19.2
Ampex 499	1.47	2.84	1.93	0.72	2.04	16.5	-4.4	-9.7	-18.5
Genoton (homogeneous)	1.40	1.38	0.95	1.02	1.39	55 (16*)	-21 (-7*)	-32.7 (-14*)	-44.7 (-31*)

* When homogeneous tape was available (in the 1940s), it was not biased for maximum long-wavelength sensitivity. I have guessed that it was biased for a recording depth of 16 µm.

6 DISCUSSION

Unfortunately the writers of magnetic recording books still mostly give the cause of the falling wavelength response as the old and erroneous "Wallace thickness loss" (equation 1 and Fig. 4, curve "b", above). This is based on the believable (but, it turns out, incorrect) assumption that the tape magnetization is constant *vs* the depth into the coating,

But we know better now: Tjaden proved that the longitudinal magnetization is actually zero at the coating surface, increasing linearly with depth, to a maximum at the back of the coating. Thus the wavelength response falls with decreasing wavelength, first at 6 dB/octave, then later at 12 dB/octave, as given in eq (2) above, and shown in Fig. 4, curve "a", the "Tjaden thickness loss".

Finally, Bertram showed (eq 3 and Fig. 5 above) that the reversible anisotropic permeabilities of the coating cause an *increase* in the short-wavelength reproduced flux above the flux calculated in eq (2), and I have prepared a computer program [13] which calculates the "Bertram thickness loss" response taking this into account.

The practical problems with the Bertram loss are that it requires a calculation or measurement of the head-to-tape spacing, and there is no known way to do that; and that the reversible permeability values for newer tapes are not usually available.

Since the measured response according to the Tjaden formula of Eq (2) and Fig 3 is very close to the the calculated value, I suspect that the increased short-wavelength response due to the permeability, is very similar to the decreased response due to the spacing loss. Thus, for practical purposes, one should just use the Tjaden response of Eq (2).

One might ask "are the "Tjaden and Bertram thickness losses" "recording losses", or "reproducing losses"? The answer is that it is some of each: Most of the parameters are set at the time of recording, but the effect of the magnetization *vs* depth, and of the permeabilities, take place during reproduction. Spacing during reproduction is both set and takes place in reproduction.

In the present paper, I have merely summarized Neal Bertram's assumptions and conclusions about the wavelength response. He also discusses many other important things that I do not mention here – I think that this paper is already complicated enough.

If you want to understand all of the details, you need to read the Bertram and the Tjaden referenced papers, which are all available online.

7 REFERENCES

Note: Most of the references cited below are available online at the URLs shown.

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http://www.aes.org/e-lib/browse.cfm?elib=3057
also revised draft of 2009.

[2] J. G. McKnight, "The Frequency Response of Magnetic Recorders for Audio", *Jour. Audio Eng. Soc*, Vol. 8, pp. 146...153 (1960-07). See especially Fig. 12.
http://mrltapes.com/mcknight_freq-resp-of-mag-recorders.pdf

[3] J. G. McKnight, "Flux and Flux-Frequency Measurements and Standardization in Magnetic Recording", *Jour. SMPTE*, Vol. 78, pp. 457...472 (1969-06).
http://mrltapes.com/mcknight_flux-and-flux-frequency-response-measurements.pdf

[4] International Electrotechnical Commission, Standard 60 094-1 "Magnetic tape sound recording and reproducing systems, Part 1, General conditions and requirements", 4th ed, 1981. The standard actually defines the *flux recorded on the tape* vs frequency, which falls 6 dB/octave above a standardized "transition frequency", then the standard specifies that the reproducer will have the inverse (rising) response in order to make the overall frequency response flat.
http://webstore.iec.ch/webstore/webstore.nsf/mysearchajax?Openform&key=iec%2060094&sorting=&start=1&onglet=1
The transition frequencies are also given in Table 1 on page 3 of "Choosing and Using MRL Calibration Tapes...", http://mrltapes.com/choo&u.pdf

[5] R. L. Wallace, Jr, "The Reproduction of Magnetically Recorded Signals", *Bell System Tech J.* 30, 1145...1173 (1951). Reprinted in M. Camras (ed), "Magnetic Tape Recording", Van Nostrand, 1985.
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[10] E. D. Daniel and Levine, “Determination of the Recording Performance of a Tape from Its Magnetic Properties”, *J Audio Eng Soc*, Vol. 7:4, 181...188, 222; (1959 Oct). http://www.aes.org/e-lib/browse.cfm?elib=547

[11] H. N. Bertram, “Wavelength Response in AC Biased Recording”, *IEEE Trans Magnetics*, Vol MAG-11:5, pp 1176...1178 (1975-09). http://mrltapes.com/bertram_wavelength-response.pdf

[12] H. N. Bertram, “Anisotropic Reversible Permeability Effects in the Magnetic Reproduce Process”, *IEEE Trans Magnetics*, Vol MAG-14:3, pp 111...118 (1978-05). http://mrltapes.com/bertram_anisotropic.pdf

[13] The DOS program is at http://mrltapes.com/tapefr.exe , with an explanation that is readable with any text editor at http://mrltapes.com/tapefr.seq . (NOTE: Those programs have an obsolete address and URL for MRL. Also: I have not re-done these programs for Windows, and the printing function does not work in Windows. The work-around is to use the “Print Screen” key to copy the displayed result to the clipboard, and import this to Word Pad to print. I hope to re-program these in a Windows language, such as “Excel”). For now, you can use “vDOS”, a virtual DOS that runs on Windows 10 systems.

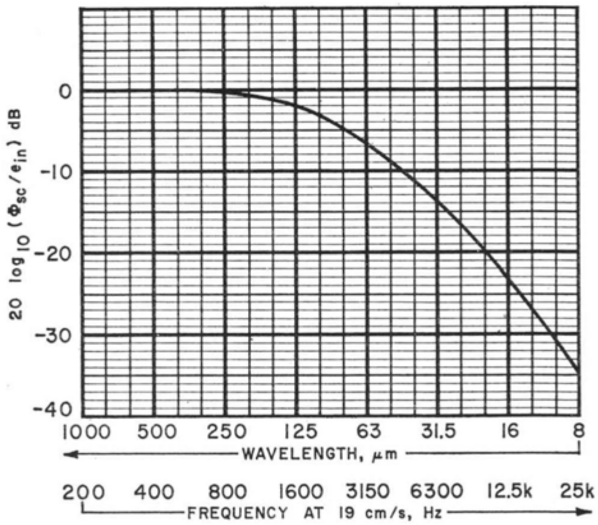

Fig 1 A measured wavelength response (upper scale) of a typical mastering tape, and the corresponding frequency response at 190 mm/s (7.5 in/s) . From McKnight [3], Fig 6.

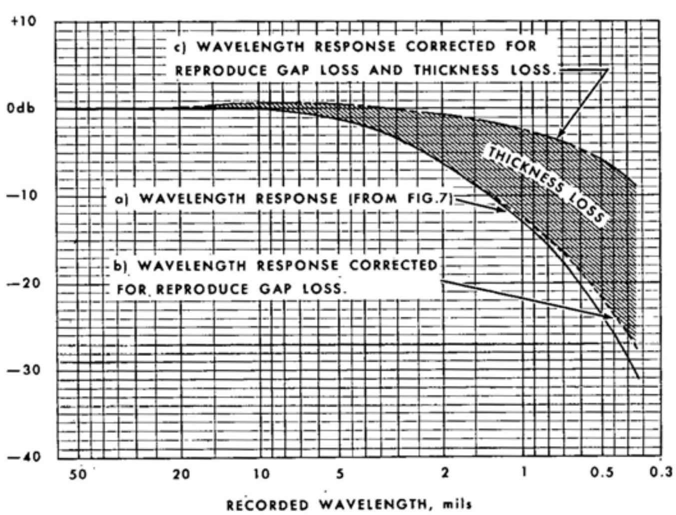

Fig 2 To verify the Wallace "thickness loss", I measured the wavelength loss (a), then corrected the response for the gap-loss of the reproducing head used (b). When I subtracted the calculated "thickness loss" (c), the response should have been "flat", but in fact there was a 9 dB loss at the 10 μm (0.375 mil) wavelength (20 kHz at 190 mm/s. From McKnight [2], Fig 12.

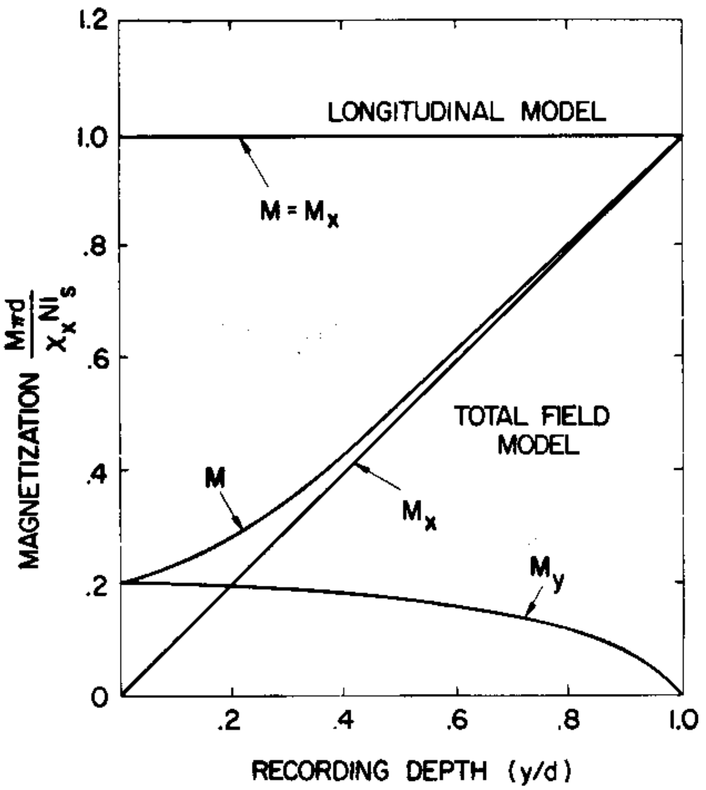

Fig 3 Recorded magnetization *vs* depth into the coating, where 0 is the tape surface, and 1.0 is the back of the coating. Wallace [2] uses the "Longitudinal Model" (top line), where the magnetization *M* is constant vs depth into the tape. Tjaden [7], and later Bertram [5] use the "Total Field Model" (bottom three curves), where the magnetization *M* is zero at the surface of the tape (depth = 0), rising to the maximum at the back of the coating (depth = 1.0). From Bertram [9], Fig 9.

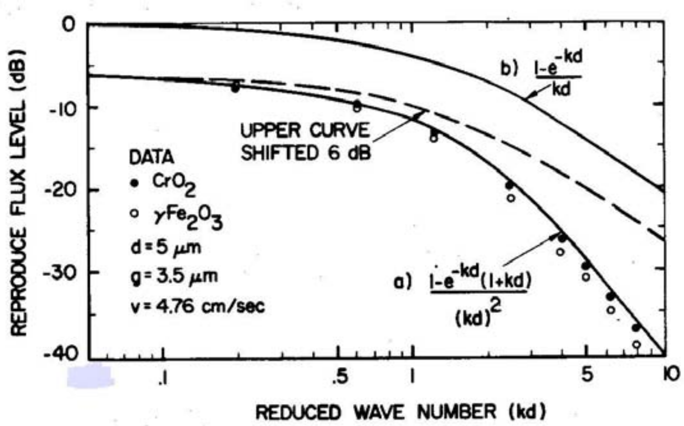

Fig 4 Comparison of measured and calculated "thickness" responses:

Circles and dots: The measured wavelength response.

b) W(*kd*), the calculated thickness response using the Wallace assumption of constant magnetization with depth. This calculation does not follow the measured response.

a) T(*kd*), the calculated thickness response using Tjaden's "total field model", which assumes linearly increasing magnetization with depth. This calculation follows the data very closely.

(From Bertram [11], Fig. 2.)

$$B(kd) = \frac{[\beta + \tanh k\alpha d - (\beta + k\alpha d)\,\mathrm{sech}\, k\alpha d\,]}{(k\alpha d)^2 \cosh ka\,[\,\beta\,(1 + \tanh ka) + (\beta^2 \tanh ka + 1)\tanh k\alpha d\,]} \quad (3)$$

Note: α is Greek "alpha". a is Roman a, NOT alpha.

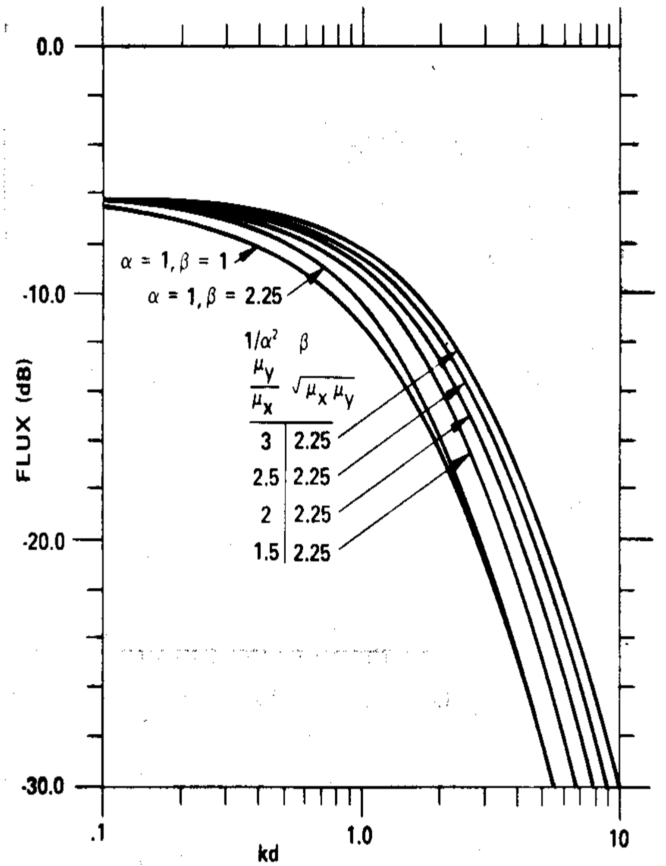

Fig 5 The function B(*kd*), which includes the effect in reproduction of the tape reversible permeabilities in the longitudinal direction μ_x, and in the vertical direction, μ_y. Fig 4 assumed permeabilities of 1 (the lower curve above). The next curve above is for a mean permeability β = 2.25 , typical of modern tapes. The next four curves are for increasing ratios of μ_y / μ_x . From Bertram [12] Fig 3.