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Loudspeaker Damping

ALBERT PREISMAN*

Part 1. A discussion of theoretical considerations of loudspeaker characteristics, together with a practical method of determining the constants of the unit as a preliminary step in obtaining satisfactory performance.

ONE OF THE CONSIDERATIONS in the design and application of loudspeakers is the adequate damping of their motion. Thus, owing to the masses and compliances involved, the sudden application or removal of current in the voice coil tends to produce a transient oscillation of a damped sinusoidal nature.

In particular, the sudden cessation of current in the voice coil may find the loudspeaker continuing to vibrate in the manner described, so that the sound "hangs over". Any one who has experienced this unpleasant effect will seek ways and means to eliminate it.

In the case of a horn type loudspeaker, the horn imposes in general sufficient mechanical loading to damp out such transient response of "hangover", and also serves to limit the excursions of the voice coil so that it does not operate into the nonlinear portion of the air-gap magnetic field. The damping also serves to minimize nonlinear compliance of the suspension system by limiting the amplitude of oscillation.

However, if the horn design is limited by such considerations as maximum permissible mouth area and is operated at a frequency not too low to be transmitted by the horn taper yet low enough so that appreciable reflections occur at the mouth, then the horn may cease to act as a mechanical resistance, but instead become predominantly reactive, and thereupon cease to damp a resonance in the speaker unit occurring in this frequency range. In such an event other means of damping will be of value to the designer or applications engineer.

In the case of the direct-radiator loudspeaker unit, the air load is small, and is mainly reactive at the lower frequencies. Hence mechanical damping of the unit is small in magnitude, and "hangover" effects may be particularly noticeable.

A reflexed cabinet may help to load the loudspeaker, or at any rate to produce a two-mesh mechanical network exhibiting two resonance peaks, neither of which is as high as that of the unit by itself or in a flat baffle. Nevertheless, the damping may still not be sufficient to produce "clean" low-frequency tones.

Hence, in general, it is advisable or at least desirable to provide sufficient damping of the direct-radiator type of unit by means of its electrical characteristics, so that whether it is operated into a horn, reflexed cabinet, or simply a flat baffle, it will be adequately damped.

An important point about electrical damping is that it represents high rather than low efficiency of operation, just as a horn does. On the other hand, were some material such as viscaloid employed to provide the required damping, the electrical input power would in part at least be converted into heat energy in the material instead of into acoustic energy, and thus represent a decrease in efficiency. It will therefore be of interest to examine damping produced by the electrical characteristics of the system.

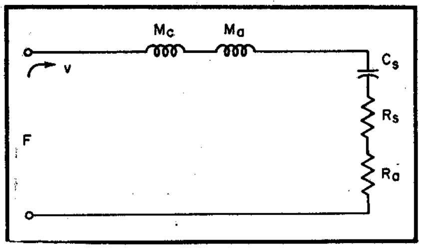

Fig. 1. Equivalent circuit of loudspeaker unit at low frequencies.

Motional Impedance

When an alternating current flows in a voice coil, it reacts with the constant magnetic field to produce an alternating force which causes the voice coil to vibrate at the frequency of the current. In so doing, the voice coil cuts through the magnetic lines, and generates a counter electromotive force, c.e.m.f.

The action is exactly similar to that of the rotating armature of a d.c. motor—the armature generates a c.e.m.f. by its rotation in the magnetic field. Consider the case of the loudspeaker voice coil. The electrical c.e.m.f. which is generated, tends to oppose the flow of current in the coil, just as if its impedance had gone up. After all, one ohm of impedance simply means a one volt drop in the unit for a one-ampere current flowing through it; i.e., volts per ampere. In the case of the loudspeaker, the force, and hence motion and c.e.m.f., are proportional to the voice coil current, so that a ratio is involved which is an apparent impedance.

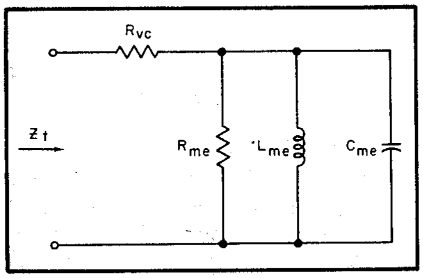

Fig. 2. Mechanical characteristics of speaker as seen from voice-coil terminals.

Hence, when a loudspeaker voice coil

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is permitted to vibrate, its impedan apparently goes up. The increase in tl impedance owing to its motion is know as the MOTIONAL IMPEDANCE, and it measured in ohms just as the electric impedance of the voice coil is measur in ohms.

Several characteristics of the motion impedance can be readily analyzed qual tatively. In the first place, the lower tl mechanical impedance, the more readi does the voice coil vibrate, and tl higher is the induced c.e.m.f. for a giv current flowing through it; i.e., tl higher is its motional impedance.

A second point to note is that tl greater the magnetic flux density, tl greater is the induced c.e.m.f., and tl higher is the motional impedance of tl voice coil. Finally, we note that if tl total length of voice-coil wire is i creased, there is more conductor cuttir the magnetic field, and hence mo c.e.m.f. induced. Therefore the motion impedance increases if the length voice coil wire is increased.

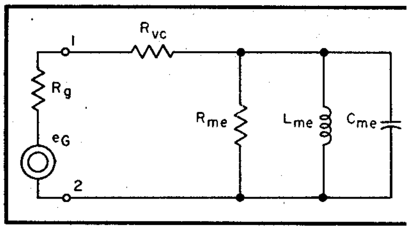

Fig. 3. Circuit of Fig. 2 with additi of generator.

The actual quantitative relations a as follows:

$$Z_{me} = \frac{(Bl)^2 \times 10^{-9}}{Z_m} \qquad (\;$$

where Z_{me} is the motional impedance electrical ohms; B is the magnetic fl density in gauss; l = length of voice c conductor in cm., and Z_m is the mecha ical impedance in mechanical oh (dynes/cm/sec.).

Loudspeaker Low-Frequency Resonance

The mechanical impedance Z_m of tl loud speaker unit varies considerab over the frequency range. However, a direct radiator its value and effect the lowest audio frequencies is of grea est importance, particularly with rega to "hangover' effects, and hence will analyzed at this point.

At the lowest audio frequencies, tl loudspeaker unit acts mechanically as simple series resonant circuit. This illustrated in *Fig.* 1. The masses i volved are those of the cone, M_c, and the air set in motion by the cone M The latter is a function of frequenc but can be assumed fairly constant ov

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a narrow frequency range involving the resonant frequency of the unit.

The compliance C_s represents that of the suspension, both of the rim of the cone and of the center spider. It is apt to be nonlinear, particularly for large excursions, but is reasonably constant for moderate and small amplitudes of vibration.

The resistive factors are that of the suspension R_s, and that of the air set in motion by the cone, R_a. The latter is particularly variable with frequency, but is usually very small at the low frequency at which resonance occurs, particularly if the speaker unit is tested by itself, or at most in a flat baffle. Values for several sizes of cones are given by Olson.[1]

From *Fig.* 1, it is apparent that

$$Z_m = (R_s + R_a) + j\omega(M_c + M_a) + 1/j\omega C_s \qquad (2)$$

Substituting this in Eq. (*1*), we obtain

$$Z_{me} = \frac{(Bl)^2 \times 10^{-9}}{(R_s + R_a) + j\omega(M_c + M_a) + 1/j\omega C_s} \qquad (3)$$

If we divide the numerator and denominator of the right side of Eq. (*3*) by $(Bl)^2 \times 10^{-9}$ we obtain

$$Z_{me} = \frac{1}{\dfrac{(R_s + R_a)}{(Bl)^2 \times 10^{-9}} + j\omega\dfrac{(M_c + M_a)}{(Bl)^2 \times 10^{-9}} + \dfrac{1}{j\omega C_s (Bl)^2 \times 10^{-9}}} \qquad (4)$$

Let

$$(R_s + R_a)/(Bl)^2 \times 10^{-9} = G_{me} = 1/R_{me}$$
$$(M_c + M_a)/(Bl)^2 \times 10^{-9} = C_{me}$$
$$\text{and } C_s(Bl)^2 \times 10^{-9} = L_{me} \qquad (5)$$

where

R_{me} is the motional resistance corresponding to the mechanical damping R_s and R_a,

C_{me} is the motional capacitance corresponding to M_c and M_a, and

L_{me} is the motional inductance corresponding to C_s.

In short, we shall assume that the mechanical resistance appears as an electrical conductance $G_{me} = 1/R_{me}$; the mechanical compliance appears as an electrical inductance; and the mechanical mass appears as an electrical capacitance. The latter transformation has been known for a long time in the power field; years ago oscillating synchronous motors were used in Europe as electrical capacitors, since a relatively small armature mass appeared as a surprisingly large electrical capacitance.

If we substitute Eq. (*5*) in Eq. (*4*), we obtain:

$$Z_{me} = \frac{1}{(1/R_{me}) + j\omega C_{me} + (1/j\omega L_{me})} \qquad (6)$$

The quantities on the right side represent a resistance, capacitance, and inductance *in parallel*, since the parallel impedance is equal to the reciprocal of the sum of the reciprocals of the individual impedances.

Hence we finally arrive at the conclusion that the mechanical characteristics of the loudspeaker at the lower frequencies appear at the electrical terminals of the voice coil as shown in *Fig.* 2. Here R_{vc} represents the electrical resistance of the voice coil; the electrical (clamped) inductance of the voice coil can be disregarded at the lower audio frequencies.

The mechanical characteristics of the speaker appear as a parallel resonant circuit shunted by a certain amount of resistance; these constitute the motional impedance Z_{me} of the speaker, and the

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[1] H. F. Olson, "Elements of Acoustical Engineering," p. 126. D. Van Nostrand Co., New York.

total electrical impedance Z_t is Z_{me} plus R_{vc}.

We can now analyze the behavior of the speaker from its electrical motional impedance characteristics. Thus, just as *Fig.* 1 indicated a certain frequency of resonance, so does *Fig.* 2 indicate this fact. Since the two circuits are equivalent, they must have the same resonant frequency. This can be readily shown. Thus, from Eq. (*5*)

$$L_{me}\,C_{me} = C_s\,(Bl)^2 \times 10^{-9} \frac{(M_c + M_a)}{(Bl)^2 \times 10^{-9}}$$
$$= (M_c + M_a)C_s \qquad (7)$$

that is, the electrical *LC* product equals the mechanical *MC* product; either therefore represents the same resonant frequency.

It will be of interest to compare the behavior of the electrical circuit of *Fig.* 2. For example, at the resonant frequency of the loudspeaker, namely

$$f_r = \frac{1}{2\,\pi(M_c + M_a)C_s} = \frac{1}{2\,\pi\,L_{me}\,C_{me}} \qquad (8)$$

the mechanical current or velocity v is a maximum, and is in phase with the force F, *Fig.* 1.

This in turn means that the electrical c.e.m.f. will be a maximum and in phase opposition with the force F, which in turn is in phase with the current in the voice coil. Hence this c.e.m.f. will produce an in-phase or *resistive* reaction: the generator will view the voice coil as having increased in impedance, and that this increased impedance is resistive in nature.

Now refer to *Fig.* 2. At the frequency of resonance, L_{me} and C_{me} act as an open circuit shunting R_{me}, so that the electrical impedance is

$$Z_t = R_{vc} + R_{me} \qquad (9)$$

and is a maximum. Furthermore, if the mechanical resistance $(R_s + R_a)$ is small, v will be a maximum, as will also be the c.e.m.f., whereupon the electrical source will see a *high* resistive impedance R_{me}. This checks the inverse relation between R_{me} and $(R_s + R_a)$ given in Eq. (*5*); when $(R_s + R_a)$ is small, R_{me} appears large since $(R_s + R_a)$ appears in the denominator of the expression for R_{me} in Eq. (*5*).

To be concluded in the April issue.

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Loudspeaker Damping

ALBERT PREISMAN*

Part 2. A discussion of theoretical considerations of loudspeaker characteristics, together with a practical method of determining the constants of the unit as a preliminary step in obtaining satisfactory performance.

WE NOW COME to the question of damping of the loudspeaker mechanism by the electrical circuit. In *Fig* 3 is shown the electrical equivalent of a loudspeaker illustrated in *Fig.* 2, with the addition of an electrical source of internal resistance R_G feeding it. This normally represents the R_p of the output tube or tubes as viewed from the secondary terminals of the output transformer.

The apparent generated voltage as viewed from the secondary terminals is e_G. The transient solution, however, is that current which flows in the network when e_G is zero, and subject to whatever initial conditions we seek to impose.

This circuit has been solved innumerable times; the current flow is oscillatory in nature, and of a frequency and decrement determined by the *L, C,* and *R* of the circuit. In particular, if

$$R = \sqrt{L_{me}/C_{me}}$$

$$= \frac{1}{2\pi f_r C_{me}} \qquad (10)$$

where f_r is given by Eq. (*8*), and R is the resistance paralleling L_{me} and C_{me}, then the circuit is critically damped. This means that the natural frequency is zero, or the circuit is no longer oscillatory; physically the loudspeaker has no hangover effect. Of course R can be less than the value given by Eq. (*10*); the latter merely gives the maximum permissible value of R.

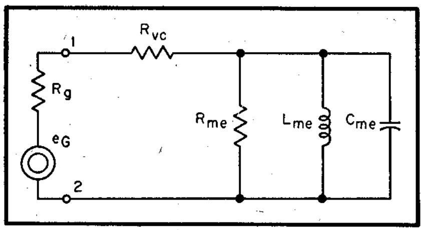

Fig. 3. Circuit of Fig. 2 with addition of generator.

An inspection of *Fig.* 3. indicates that R must represent R_{me} paralleled by $(R_{vc}+R_G)$, hence if R_{me} is greater than the value required by Eq. (*10*), $(R_{vc}+R_G)$ must be a low enough shunt to provide in conjunction with R_{me} the critical damping necessary.

It will be recalled from Eq. (*5*) that if the mechanical damping (R_s+R_a) is low, R_{me} will be correspondingly high. An example which is to follow will show that usually the mechanical damping (R_s+R_a) is very low, so that it can be expected that R_{me} will be relatively very high; much higher than will provide critical damping.

From this it follows that $(R_{vc}+R_G)$ must be a sufficiently low shunt to satisfy the critical damping condition given by Eq. (*10*). However, it is possible that the voice coil resistance R_{vc} is itself so high that Eq. (*10*) cannot be satisfied. In the usual case R_{vc} is not too high, but the maximum value left for R_G to assume can be quite low. In such a case a large amount of inverse voltage feedback may be necessary to reduce the source impedance to the requisite low value.

Numerical Example

The following numerical example will serve to illustrate the above analysis. Suppose we take a 16-inch cone type loudspeaker, whose mass is 40 grams, plus 4 grams for the voice coil. Assume further that the compliance of the suspension is $C_s = 3.2 \times 10^{-7}$ cm/dyne, and that the mechanical resistance is 2400 mechanical ohms.

To the mass of the cone and voice coil must be added that of the mass of the air. In the neighborhood of 25 cps or so, Olson[2] gives the reactance of the air load as 7500 mechanical ohms. The corresponding mass is

$$M_a = \frac{7500}{2\pi \times 25} = 48 \text{ grams}$$

Hence the total mass is

$$M_t = 40 + 4 + 48 = 92 \text{ grams}$$

The resonant frequency is, by Eq. (*8*)

$$f_r = \frac{1}{2\pi\, 92 \times 3.2 \times 10^{-7}} = 29.3 \text{ cps}$$

which is close to the value of 25 cps initially used to calculate the air mass.

The air also imposes a certain amount of damping in the form of radiation resistance. This is a rapidly varying function of frequency; from Olson's book we find it to be 600 mechanical ohms at 29 cps. Hence the total mechanical damping is

$$R_s + R_a = 2400 + 600 = 3000 \text{ mech. ohms.}$$

Now suppose the flux density B is 10,000 gauss, and the length l of voice coil conductor is 1500 cm. Assume further that the voice coil resistance R_{vc} is 10 ohms.

Then, from Eq. (*5*), we have

$$R_{me} = \frac{(1500 \times 10^4)^2 \times 10^{-9}}{3000} = 75 \text{ ohms}$$

$$C_{me} = \frac{92}{(1500 \times 10^4)^2 \times 10^{-9}} = 409\ \mu f$$

$$L_{me} = (3.2 \times 10^{-7})\ (1500 \times 10^4)^2 \times 10^{-9}$$
$$= 0.072 \text{ henry}$$

Observe how large C_{me} is even though the mass responsible for this capacitive effect is only 92 grams.

For critical damping, the total resist-

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[2] *Loc. cit.*

ance shunting L_{me} and C_{me} must be, by Eq. *(10)*:

$$R = \sqrt{\frac{.072}{409 \times 10^{-6}}} = 13.3 \text{ ohms}$$

Since R_{me} is one branch in parallel with R_{vc} plus the generator resistance, and this all totals 13.3 ohms, the voice coil branch must be

$$R_e = \frac{R_{me} \times R}{R_{me} - R} = \frac{75 \times 13.3}{75 - 13.3} = 16.18 \text{ ohms}$$

Since the voice coil resistance R_{vc} is 10 ohms, the generator or source resistance, as viewed from the secondary terminals of the output transformer, must be

$$R_G = 16.18 - 10 = 6.18 \text{ ohms.}$$

Although this is a low value, it is by no means prohibitively low. For example, if in the case of a single-ended triode output stage, $R_L = 2R_p$, then at the secondary terminals R_p should reflect as half of the voice coil load, if R_{vc} is 10 ohms, the reflected tube resistance R_G would be $10/2 = 5$ ohms. In short, a triode tube may be expected to act as critical damping in conjunction with the voice coil resistance.

In the case of a pentode tube, R_p is so high that no damping can be expected from it unless inverse voltage feedback is employed to an extent sufficient to lower the apparent source resistance to the required degree.

However, note that all this depends upon how low R_{vc} is compared to the length of wire used, and also how high the flux density B is. If the product (Bl) is low, both R_{me} and R may come out so low that R_{vc} alone may be in excess of that which paralleling R_{me}, will give the required value of R for critical damping. This means that even if the source resistance is zero, R_{vc} is too large and will not permit critical damping to be obtained.

Experimental Determination of Circuit Constants

It is possible to measure the motional impedance by simple electrical means, and from these measurements to determine the critical damping required. Since the measurements are to be made at the very low audio frequencies, ordinary iron vane meters can be used if so desired, and even a d.c. measurement of the voice coil resistance should be sufficient to furnish the value of R_{vc}.

If, however, it is desired to determine this quantity at the resonant frequency of the cone, or at any rate at some a.c. frequency, then the cone should be clamped so that it does not vibrate and generate a c.e.m.f., thereby furnishing a motional impedance value.

To measure the motional impedance, a set-up such as that indicated in *Fig.* 4 can be used. The audio oscillator wave shape should be reasonably free of harmonics, and the audio amplifier should be capable of furnishing several watts of power without distorting. The ammeter can be of the iron-vane type, and should read one ampere or less at full scale. The voltmeter is preferably of a high-impedance type. A preliminary run should be made to determine the resonant frequency of the cone and its suspension. This is done by varying the frequency upward in steps starting from say, 20 cps, and noting E and I at each step. Their quotient is the impedance seen looking into the voice coil. This should be done with the field fully energized if it is of the electrodynamic type.

At the mechanical series resonant frequency of the cone, I will drop to a very low value, and E will tend to rise. In short, the quotient will be relatively large, and will represent $(R_{vc} + R_{me})$. If the value found previously for R_{vc} is subtracted from this reading, R_{me} is obtained. The resonant peak is normally quite sharp for reasons that will be explained further on.

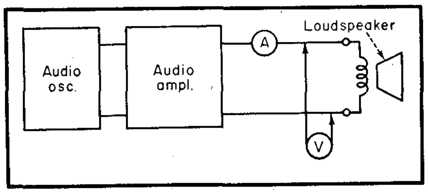

Fig. 4. Circuit arrangement used for making measurement of motional impedance.

In order to determine the value of critical damping R, it would appear necessary to measure L_{me} and C_{me}. However, R can also be determined by measuring the Q of the circuit; critical damping is obtained if $Q = 1$. To measure Q, ordinarily one merely has to plot the selectivity curve for the device, whether this curve represents transmission, impedance, admittance, or whatever other quantity gives this characteristic.

In the case of the loudspeaker, the resonant Q of the circuit is determined by the impedance as measured across R_{me}, L_{me}, and C_{me} in *Fig.* 3, with the electrical resistance $(R_{vc} + R_G)$ in parallel with R_{me}. In other words, the condition given by Eq. *(10)* for critical damping is also the condition for the resonant Q to be unity, where Q_r is in general determined by $\omega_r C_{me}$, and R_{me} and $(R_{vc} + R_G)$ in parallel.

Unfortunately, measurements must be made at terminals 1–2 in *Fig.* 3, since there are no accessible terminals across Z_{me}. The resulting impedance, Z_t, represents R_{vc} in series with Z_{me}, that is—with R_{me}, C_{me}, and L_{me} all in parallel. To find the above-defined resonant Q therefore requires some preliminary analysis, which will be given below.

Experimentally, however, all one has to do is to measure the impedance Z_t at and around resonance over a range including frequencies at which Z_t drops to $1/\sqrt{2}$ of its value at resonance (where it has the maximum value $R_{vc} + R_{me}$). Then, knowing the two frequencies at which this occurs, as well as the resonant frequency f_r, Q can be calculated. Once Q is known, the necessary value of R can be found, and then the maximum permissable generator resistance R_G.

Let us therefore proceed to evaluate this impedance. The impedance looking to the right into terminals 1–2 of *Fig.* 3 can be calculated from the circuit elements shown. It is:

$$|Z_t| = \sqrt{\frac{1 + Q_r^2\,(1-\rho^2)^2}{\left(\frac{1}{R_{me}+R_{vc}}\right)^2 + \frac{Q_r^2}{R_{vc}^2}\,(1-\rho^2)^2}} \qquad (11)$$

where Q_r is the resonant Q of the circuit *if terminals 1–2 of Fig. 3 were short-circuited;* i.e.,

$$Q_r = \omega_r C_{me} R = R/\omega_r L_{me} \qquad (12)$$

in which R represents R_{me} and R_{vc} in parallel, and ω_r is the resonant angular velocity of L_{me} and C_{me}. Furthermore,

$$\rho = f/f_r \qquad (13)$$

where f is the frequency at which Z_t is being measured, and f_r is the resonant frequency; in short, ρ represents the fractional deviation from the resonant frequency.

In particular, if $\rho = 1$, $(f = f_r)$, Eq. *(11)* reduces to

$$Z_t = R_{me} + R_{vc} \qquad (14)$$

which is correct from an inspection of *Fig.* 3, since at the resonant frequency L_{me} and C_{me} form a negligibly high shunt impedance across R_{me}, so that Z_t becomes $R_{me} + R_{vc}$, as stated above.

Furthermore, if $\rho = 0$, $(f = 0)$, or $\rho = \infty$, $(f = \infty)$, Z_t becomes equal to R_{vc} alone, as is also clear from *Fig.* 3, since L_{me} is a short circuit across R_{me} at $f = 0$, and C_{me} is the short circuit at $f = \infty$.

If Eq. *(11)* is solved for Q_r in terms of the other variables, there is obtained:

$$Q = \frac{1}{(1-\rho^2)}\sqrt{\frac{\left(1 - \frac{Z_t}{R_{me}+R_{vc}}\right)^2}{(Z_t/R_{vc})^2 - 1}} \qquad (15)$$

Now suppose the frequency is varied, which is the same as saying ρ is varied until Z_t drops to $1/\sqrt{2}$ of its maximum value; i.e.,

$$Z_t = \frac{R_{me} + R_{vc}}{\sqrt{2}} \qquad (16)$$

If this value is substituted in Eq. *(15)*, together with the corresponding specific

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value of ρ, call it ρ_1, there is obtained:

$$Q_r = \left(\frac{1}{1-\rho_1{}^2}\right)\sqrt{\frac{1}{\left(\frac{R_{me}+R_{vc}}{R_{vc}}\right)^2 - 2}} \qquad (17)$$

If $\left(\frac{R_{me}+R_{vc}}{R_{vc}}\right)^2 \gg 2$, say twenty times two, then Eq. *(17)* simplifies to

$$Q_r = \left(\frac{1}{1-\rho_1{}^2}\right)\left(\frac{R_{vc}}{R_{me}+R_{vc}}\right) \qquad (18)$$

If ρ_1 is nearly unity, the difference between the actual frequency f_1 and the resonant frequency f_r is small; that is,

$$\Delta f_1 = f_r - f_1$$

or

$$\Delta f_1 = f_1 - f_r$$

(depending upon whether the excursion is below or above the resonant frequency) is small. This is usually the case, and under such conditions Eq. *(18)* can be rewritten as

$$Q_r = \left(\frac{f_r}{2\Delta f_1}\right)\left(\frac{R_{vc}}{R_{me}+R_{vc}}\right) \qquad (19)$$

which can form the basis of our experimental procedure as well as Eq. *(18)* can. If we re-write Eq. *(19)* as follows:

$$\frac{2\Delta f_1}{f_r} = \left(\frac{1}{Q_r}\right)\left(\frac{R_{vc}}{R_{me}+R_{vc}}\right) \qquad (20)$$

we recognize the form to be similar to that of the well-known resonance formula, in which the fractional bandwidth $(2\Delta f/f_r)$ for the half-power points is the reciprocal of the resonant Q of the circuit. Eq. *(20)* shows that owing to the point in the circuit at which the measuring instruments are introduced, the fractional bandwidth is reduced by a factor $R_{vc}/(R_{me}+R_{vc})$, which would not occur if the measurements could be made across the motional impedance component itself.

The significance of Eq. *(20)* is that even though Q_r for a loudspeaker system may be less than unity, the fractional bandwidth will nevertheless be quite small because of the reducing factor $R_{vc}/(R_{me}+R_{vc})$. This makes the measurements somewhat critical and requires a well-calibrated frequency scale on the audio oscillator.

To see how this all fits together, let us proceed with an experimental run. The first measurement is R_{vc}; this is found to be 10 ohms. Then the test setup of *Fig.* 4 is connected to the loudspeaker and the frequency varied from say 20 to 50 cps.

At 29.3 cps the current is found to dip to a minimum value of 83.2 ma, and the voltmeter reads 7.07 volts. The impedance is resistive, and of a value

$$R_{vc} + R_{me} = 7.07/.0832 = 85 \text{ ohms.}$$

Hence $Z_t = R_{me} = 85 - 10 = 75$ *ohms.*

Now the frequency is varied above and below 29.3 cps to the point where Z_t drops to $85\sqrt{2} = 60.1$ ohms, as found by taking the ratio of the voltmeter to ammeter readings in exactly the same way as $(R_{vc}+R_{me})$ was calculated.

Suppose the frequency drops from 29.3 to 26.7 cps before $Z_t = 60.1$, and rises to 31.9 cps before this value is reached once more. Then $\Delta f_1 = 29.3 - 26.7 = 2.6$ cps, or $\Delta f_1 = 31.9 - 29.3 = 2.6$ cps, and

$$2\Delta f_1/f_1 = 2 \times 2.6/29.3 = 0.1776.$$

We can now use Eq. *(19)* to calculate Q_r. Thus

$$Q_r = \left(\frac{1}{0.1776}\right)\left(\frac{10}{85}\right) = 0.663.$$

This is the Q of the loudspeaker circuit if the source impedance R_G were zero. Since Q_r is less than unity, it can be raised to that figure by allowing R_G to be greater than zero. It remains to calculate this value.

We have, for a parallel resonant circuit such as in *Fig.* 3, that

$$Q = \omega_r C_{me} R \qquad (21)$$

where R is the resistance shunting C_{me} and L_{me} (*Fig.* 3), and is therefore R_{me} in parallel with $(R_{vc}+R_G)$. However, in the measurement and calculation yielding Q_r, R_G is essentially zero, and R represents simply R_{me} and R_{vc} in parallel.

We seek a value R', such that the Q is equal to unity; i.e.,

$$1 = \omega_r C_{me} R'$$

or

$$R' = 1/\omega_r C_{me} \qquad (22)$$

Substituting from Eqs. *(21)* and *(20)* in Eq. *(22)*, we obtain

$$R' = \frac{R}{Q_r} = \frac{R_{me}R_{vc}}{R_{me}-R_{vc}} \times \frac{2\Delta f_1}{f_r} \times \frac{R_{me}+R_{vc}}{R_{vc}} = \frac{2\Delta f_1}{f_r} R_{me} \qquad (23)$$

This represents R_{me} paralleled by $(R_{vc}+R_G)$, hence

$$R_{vc} + R_G = \frac{R' R_{me}}{R_{me} - R'} \qquad (24)$$

and

$$R_G = \frac{R'(R_{me}+R_{vc}) - R_{vc}R_{me}}{R_{me} - R'} \qquad (25)$$

Hence let us finish our experimental determination of R_G. From Eq. *(23)* we can find R'. If we use the last form, we have

$$R' = \frac{2\Delta f_1}{f_r} R_{me} = (0.1776)\ (75) = 13.31$$

ohms and from Eq. *(25)* we obtain

$$R_G = \frac{(13.31)\ (85) - (10)\ (75)}{(75 - 13.31)} = 6.19 \text{ ohms}$$

which of course checks the previous computation from the values for the mechanical constants, since it is the same loudspeaker that we have under consideration.

An Alternative Viewpoint

It is possible to reflect the electrical constants into the mechanical side of the circuit, and obtain an alternative viewpoint of the behavior of the system as a whole. The results, so far as the low-frequency resonance is concerned, are the same, as will be shown. There is, however, another advantage of this alternative point of view with regard to the acoustical design; it permits the designer to incorporate the electrical constants into the acoustical design with a corresponding improvement in the performance of the loudspeaker.

First, the design formulas will have to be presented. The electrical impedance of the source and the voice coil appears in the mechanical side of the system as follows:

$$Z_{em} = \frac{(Bl)^2 \times 10^{-9}}{Z_e} \qquad (26)$$

where Z_{em} is the mechanical impedance equivalent to the actual electrical impedance Z_e, and B and l have the same significance as before.

The output stage and voice coil in series with it exhibit essentially an inductive and resistive impedance at the higher audio frequencies. The inductance is the leakage inductance of the output transformer, plus that of the voice coil, and the resistance is the apparent source resistance R_G as viewed from the secondary terminals of the output transformer, plus that of the voice coil.

Hence, set

$$Z_e = R_e + j\omega L_e \qquad (27)$$

where $R_e = R_{vc} + R_G$ (see *Fig.* 3), and L_e is the inductance defined above, and which we have not heretofore taken into account. At the lower audio frequencies $j\omega L_e$ can be ignored, whereupon Z_e reduces to R_e.

However, if Eq. (*27*) be substituted in Eq. (*26*), and then numerator and denominator divided by $(Bl)^2 \times 10^{-9}$, as before, there is obtained:

$$Z_{em} = \frac{1}{\dfrac{R_e}{(Bl)^2\, 10^{-9}} + \dfrac{j\omega L_e}{(Bl)^2\, 10^{-9}}} \qquad (28)$$

If we consider $R_e/(Bl)^2\, 10^{-9}$ as a mechanical conductance G_{em}, so that its reciprocal R_{em}, a mechanical resistance, is given

$$R_{em} = 1/G_{em} = 1/[R_e/(Bl)^2\, 10^{-9}] \qquad (29)$$

and if we further consider $L_e/(Bl)^2\, 10^{-9}$ as a mechanical compliance C_{em}, then we can write Eq. (*28*) as

$$Z_{em} = \frac{1}{(1/R_{em}) + j\omega C_{em}} \qquad (30)$$

or the electrical resistance and inductance in series appear in the mechanical system as a mechanical resistance and compliance in parallel. Hence, the counterpart of *Fig.* 3 is that shown in *Fig.* 5: a constant-velocity mechanical generator (counterpart of a constant-voltage electrical generator) feeds the mechanical resistance R_{em} equivalent to the electrical resistance R_e, in parallel with the mechanical compliance C_{em} equivalent to the electrical inductance L_e, and the actual mechanical impedance Z_m of the loudspeaker. This circuit has interesting implications both at the low- and at the high-frequency ends of the audio spectrum.

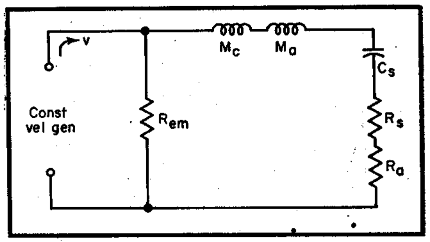

Fig. 5. Counterpart of Fig. 3, in mechanical terminology.

Consider the low-frequency end first. In this range C_{em} can be ignored, and Z_m consists, in the case of a direct-radiator cone loudspeaker, of the elements shown in *Fig.* 1. Hence *Fig.* 5 becomes circuit shown in *Fig.* 6. Here it is apparent how the electrical resistance R_e introduces in effect damping into the mechanical circuit by its transformed element R_{em}.

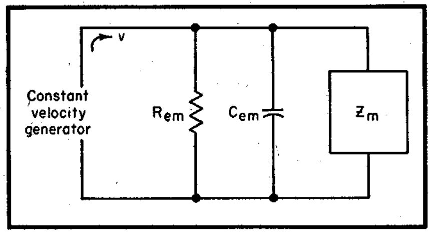

Fig. 6. Equivalent circuit corresponding to Fig. 5, showing damping due to electrical resistance.

From *Fig.* 6 it is apparent that for critical damping,

$$(R_{em} + R_s + R_a) = \sqrt{(M_c + M_a)/C_s} \qquad (31)$$

or alternatively, that the mechanical Q at resonance is unity:

$$Q_m = \frac{\omega_r (M_c + M_a)}{R_{em} + R_s + R_a} = 1 \qquad (32)$$

from which the required electrical resistance must be

$$R_{em} = \omega_r (M_c + M_a) - (R_s + R_a) \qquad (33)$$

Once R_{em} is evaluated from Eq. (*33*), the equivalent electrical resistance R_e can be found from Eq. (*29*). Then the voice coil resistance R_{vc} is subtracted from R_e to yield the maximum permissible value of apparent generator resistance R_G.

Let us try out these formulas on the loudspeaker constants given previously. It will be recalled that the total mass (including that of the voice coil) was 92 mechanical ohms. This will be the value used for $(M_c + M_a)$. The resonant frequency was 29.3 cps, so that $\omega_r = 2\pi\ 29.3$ *rad./sec.* Also $(R_s + R_a)$ came out to be 3000 mechanical ohms.

Hence, if the appropriate values be substituted in Eq. (*33*), there is obtained:

$$R_{em} = (2\pi\, 29.3)(92) - (3000) = 16{,}980 - 3{,}000 = 13{,}980 \text{ mech. ohms}$$

Now, from Eq. (*29*), the equivalent electrical resistance R_e that is required to obtain critical damping is

$$R_e = \frac{(Bl)^2\, 10^{-9}}{R_{em}} = \frac{(10000 \times 1500)^2}{13980}$$

$= 16.13$ *ohms* (*electrical*). Since the voice coil resistance R_{vc} is 10 ohms, the apparent source resistance can be

$$R_G = 16.13 - 10 = 6.13 \text{ ohms}$$

which checks our previous calculations, as it should.

High-Frequency Response

The same equivalence between circuits can be utilized in the analysis of a high-frequency tweeter unit of the horn type. This employs a small diaphragm and voice coil, which feeds the cavity in front of it that leads to an exponential horn. The physical arrangement is shown in cross-section in *Fig.* 7. Here m_d represents the mass of the diaphragm and associated voice coil; C_a, the compliance of the air chamber in front of the diaphragm, necessary to furnish clearance for the motion of the diaphragm and useful in building out the mechanical circuit; and finally r_h represents the acoustical resistance of the horn throat in the frequency range above its low-frequency cutoff point.

The mechanical circuit has been analyzed many times in the past; it is given in *Fig.* 8. The resistance r_h is that of the throat of the horn, and is equal to the area of the throat in sq. cm. multiplied by 41.4 mech. ohms, which is the radiation resistance of air per sq. cm. A_d is the area of the diaphragm; in conjunction with A_h it forms a kind of hydraulic press which is the mechanical counterpart of an electrical transformer. The step-down ratio is A_d to A_h; conversely r_h is reflected to the diaphragm as an equivalent resistance r'_h such that

$$r'_h = (A_d/A_h)^2 \qquad (34)$$

The reflected resistance r'_h shunts the air chamber compliance C_a. This is because the lower r'_h is, the more readily can it relieve the pressure built up in the air chamber by the motion of the diaphragm. This is exactly analogous to the reduction in the charge and voltage across a capacitor when it is shunted by a low resistance.

From *Fig.* 8 the loudspeaker unit is recognized as forming an L-section low-pass filter. For proper transmission up to the cut-off frequency, it is necessary that

$$r'_h = \sqrt{M_d/C_a} \qquad (35)$$

The cutoff frequency is given by

$$f_c = \frac{1}{\pi\sqrt{M_d C_a}} \qquad (36)$$

If twice the mass $(2M_d)$ were employed and another compliance C_a placed at the left end, a π-section filter would be obtained, to which Eqs. *(35)* and *(36)* would apply equally well. In short, the same cutoff frequency can be obtained *for double the mass,* if a compliance is placed at the other end of it.

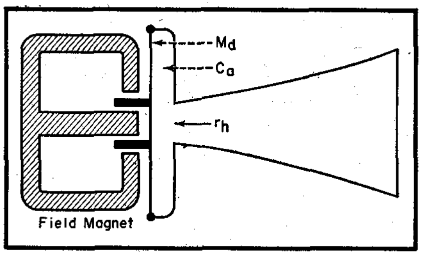

Fig. 7. Physical arrangement of mechanical elements of a high-frequency horn and unit.

If only the mass is doubled, then the cutoff frequency is reduced to 70.7 per cent of its original value, as is evident by substituting $2M_d$ for M_d in Eq. *(36)*. (The corresponding changes in r'_h are not of importance as they involve merely a change in the ratio of A_d to A_h.

For a given high-frequency cutoff and power-handling ability of the speaker, the diaphragm mass M_d comes out to be a certain amount. If M_d can be kept the same, and yet a compliance placed at the front end, the high-frequency cutoff can be extended to $\sqrt{2}$ or 1.414 times its original value without altering the speaker's power handling ability. Hence it is of interest to see how this can be done.

At the higher audio frequencies, the output transformer appears at its secondary terminals essentially as a series inductance L_L (its leakage inductance). The power amplifier tubes, as reflected to the secondary of the transformer appear as a resistance R_G in series with L_L. To this must be added the voice coil resistance R_{vc} and its inductance L_{vc} in series with R_G and L_L. Hence finally the electrical current appears as

$$Z_e = R_e + j\omega L_e$$

where

$$R_e = R_G + R_{vc} \qquad (37)$$

and

$$L_e = L_L + L_{vc}$$

Figure 5 and Eq. *(30)* show how these appear in the mechanical circuit. The mechanical impedance Z_m is in this case illustrated by *Fig.* 8, so that finally in *Fig.* 9 is given the complete mechanical circuit including the equivalent electrical circuit parameters.

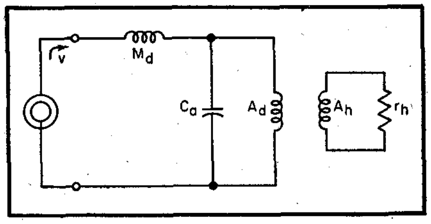

Fig. 8. Mechanical circuit of high-frequency speaker.

Here, in accordance with Eq. *(29)*

$$R_{em} = \frac{1}{R_e/(Bl)^2\,10^{-9}} \qquad (29)$$

and this should match r'_h for maximum power transfer, or

$$R_e = (Bl)^2 \times 10^{-9}/r'_h \qquad (38)$$

from which the apparent source impedance should equal

$$R_G = R_e - R_{vc} = \frac{(Bl)^2 \times 10^{-9}}{r'_h} - R_{vc} \qquad (39)$$

The apparent mechanical compliance equivalent to the electrical inductance is indicated by Eqs. *(28)* and *(30)*, namely:

$$C_{em} = L_e/(Bl)^2 \times 10^{-9} \qquad (40)$$

However, in order to convert the L-section mechanical low-pass filter of *Fig.* 8 into the π-section low-pass filter of *Fig.* 9, it is necessary that

$$C_{em} = C_a = \frac{L_e}{(Bl)^2 \times 10^{-9}} \qquad (41)$$

If such coordination in electrical and mechanical design be accomplished, a 41 per cent increase in frequency response may be expected over the case of no electrical inductance at all. Of course, in actual practice the electrical system inherently has inductance and resistance so that the "building-out" of the L-section into a π-section tends to take place; all that it is desired to point out here is that the electrical and mechanical circuit elements can be coordinated so as to improve the performance rather than to have a haphazard relationship to one another, and that furthermore, electrical inductance is not necessarily an undesirable characteristic in the output stage, but can serve a useful purpose.

Undoubtedly, in most systems the inductance—particularly that of the voice coil itself—is too high and produces a C_{em} in excess of C_a. Also, R_{em} may be too low compared to r'_h because of excessive electrical resistance $R_G + R_{vc}$. However, this serves to counterbalance an excessive value for C_{em} and therefore tends to smooth out the response.

The interested experimenter can calculate the actual response of the network shown in *Fig.* 9 on the basis that it is not a truly terminated low-pass filter section, since a resistance such as r'_h is but a nominal match over the pass band, and is a considerable mismatch near the cutoff frequency, where the termination should approach zero. He can also calculate the response for his actual speaker and amplifier output stage, in order to see directly the effect of varying, for example, the electrical circuit constants.

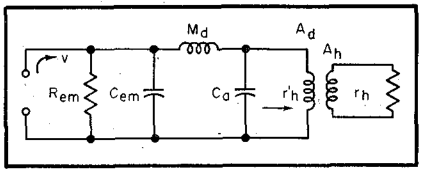

Fig. 9. Conversion of Fig. 8 circuit to pi-section equivalent.

Conclusion

A method of coordinating the motional impedance of a loudspeaker with the electrical impedance has been presented here with the object of reducing "hangover" effects and objectionable transients in general at the low-frequency resonance of the speaker.

An experimental method has also been presented to enable the necessary measurements to be made in order that the correct source impedance be obtained for critical damping of the system. The method requires merely an audio oscillator, an a-c voltmeter and an a-c ammeter in order to determine the impedance over a range of frequencies. From the shape of the impedance curve the Q of the system can be determined, and from the value of voice coil and motional impedance at resonance, the requisite source resistance for critical damping can be calculated.

An alternative method based on viewing the electrical constants from the mechanical side was then presented, and it was shown that this method led to the same answers as above. Finally, it was shown by this method how inductance and even resistance in the electrical system could be put to use to obtain a coordinated system in the case of a high-frequency loudspeaker.