Crossover Networks: A Review

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Crossover networks for multi-way loudspeaker systems and audio processing are reviewed, including both analog and digital designs. A high-quality crossover network must maintain a flat overall magnitude response, within small tolerances, and a sufficiently linear phase response. Simultaneously, the crossover filters for each band must provide a steep transition to properly separate the bands, also accounting for the frequency ranges of the drivers. Furthermore, crossover filters affect the polar response of the loudspeaker, which should vary smoothly and symmetrically in the listening window. The crossover filters should additionally be economical to implement and not cause much latency. Perceptual aspects and the inclusion of equalization in the crossover network are discussed. Various applications of crossover filters in audio engineering are explained, such as in multiband compressors and in effects processing. Several methods are compared in terms of the basic requirements and computational cost. The results lead to the recommendation of an all-pass-filter–based Linkwitz-Riley crossover network, when a computationally efficient minimum-phase solution is desired. When a linear-phase crossover network is selected, the throughput delay becomes larger than with minimum-phase filters. Digital linear-phase crossover filters having a finite impulse response may be designed by optimization and implemented efficiently using a complementary structure.

0 INTRODUCTION

A high-quality loudspeaker contains two or more drivers and a crossover network that appropriately divides the input audio signal to the different drivers according to their operating frequency range [1–4]. For example, in a two-way loudspeaker, the low frequencies are reproduced by a woofer and the high frequencies by a tweeter. To avoid distortions and damaging the drivers, the crossover filters must pass to each unit only the frequencies within its operating band. However, it is also crucial that the overall output of the multi-way loudspeaker is colorless, does not distort the signal phase much, and has controlled directivity when listened to from different angles. The crossover network affects all these characteristics. This paper reviews the theory and various designs of crossover networks.

The crossover filtering can be realized through analog or digital circuits. There are two analog alternatives: a passive crossover [5, 6], in which the filter components are placed between the power amplifier and the loudspeaker drive units, as shown in Fig. 1(a), or an active crossover [2, 4], in which the filters are placed in the line level signal circuits before the amplifier inputs that are directly connected to the corresponding drive unit(s), as presented in Fig. 1(b). The fact that each driver is connected to its own power amplifier avoids the influence of frequency-dependent variations of the input impedance that is present in passive crossovers [7].

Fig. 1(c) shows the principle of a digital crossover network and its connection with the power amplifier and the driver units. The frequency response of each driver defines the working frequency band and affects the selection of the cutoff frequency of crossover filters. The two-way crossover network reported in Fig. 1 is the simplest one, but the possibility of extending it to a multi-way crossover network has been also analyzed during the years including the

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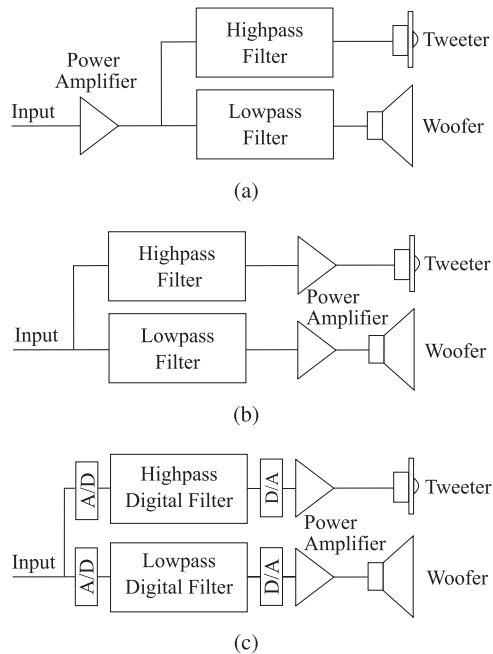
Fig. 1. Functional diagrams of (a) passive analog, (b) active analog, and (c) digital crossover networks for a two-way loudspeaker.

possibility of adding an equalization procedure to mitigate the non-ideal loudspeaker responses.

This paper provides a review on crossover network design methodologies accounting for both analog and digital filters. In particular, starting from the attributes needed for a high-quality reproduction system, the approaches of the literature are discussed following a main categorization and a historical evolution of these methodologies. The imitation of analog crossovers using infinite impulse response (IIR) digital filters is tackled, as well as the design of linear-phase crossovers, which is often based on finite impulse response (FIR) filters. Also, the need for an equalization procedure of the system in combination with the digital crossover network design will be analyzed and discussed. Additionally, perceptual aspects and other applications of crossover networks in audio processing will be reported.

This paper is organized as follows. SEC. 1 gives an overview of the historical developments on crossover networks. SEC. 2 states the principles of the loudspeaker crossover design and categorizes crossover networks based on their phase characteristics. SEC. 3 describes minimum-phase crossover networks, and SEC. 4 explains linear-phase crossover designs. SEC. 5 discusses how to combine equalization with the crossover network, and SEC. 6 reviews perceptual studies. SEC. 7 presents other applications of crossover networks. SEC. 8 compares the design methods in terms of the requirements and evaluates their computational complexity, and SEC. 9 concludes.

1 HISTORICAL DEVELOPMENTS

The limitations of individual transducers were noticed soon after the introduction of electrical sound reproduc-

tion. The earliest electrical crossover network may have been described in a German patent from the year 1924 by the Lorenz company [8]. After Kellogg and Rice had invented the dynamic cone-based loudspeaker in the mid-1920s [9], it was realized that a single large unit cannot produce frequencies higher than about 3 kHz [10, 11]. This was the starting point for developing multi-way loudspeaker systems in which two or more loudspeaker units covered a wide range of audio frequencies.

Early passive crossover networks, at the time called "dividing networks," were presented by McLachlan [12] and Hilliard and Kimball [13] in the 1930s. Hilliard and Kimball defined the "crossover frequency" as the point at which the neighboring units receive the same amount of energy [13]. At the time, the attenuation of the magnitude responses at the crossover frequency was −3 dB, and a steepness of −12 dB per octave was considered sufficient [13–15]. The recommendation was also to use as low a crossover frequency as possible, such as 200 Hz [13, 16]. The first crossover filters were based on inductors and capacitors whose component tolerances depend on the filter order [17]. Such passive crossover networks provide an affordable solution [18, 19], and they are still relevant in low-cost applications and in situations in which it is desired to avoid the power supply [20].

In 1937, Minton and Ringel patented analog filter solutions for two-way and three-way horn loudspeakers [11]. Interestingly, this historic document on crossover filters already included the idea of correcting the resonances of the loudspeaker units [11]; in other words, some degree of equalization was combined with the crossover filters. The Shearer movie theater speaker system, introduced in 1935, was an early two-way loudspeaker construction consisting of large low-frequency cone-based drivers and a multi-cellular horn for higher frequencies, with a crossover frequency of about 400 Hz [21, 22]. It soon became the norm in loudspeaker design to use at least two drivers, one for the bass and a second one for the treble sounds. The terms "tweeter" and "woofer" were already used to refer to the high-frequency and low-frequency units, respectively, in the mid-1930s [23, 24]. The term "squawker," which also comes from animal vocalization, is used at times for the midrange driver [25, 26].

During the 1970s and 1980s, active filtering became a common and practical solution replacing passive networks for several reasons [19, 27]: the ability to handle high power thanks to the direct connection between the amplifier and dedicated speaker unit, the high selectivity that can be easily achieved by simplifying the overall driver frequency response shape, the improved protection of the driver units, and, last but not least, the increasing availability of high-quality operational amplifiers used in active crossovers [28]. However, the active approach involves higher costs and complexity than the passive one, limiting its diffusion to professional and high-end applications.

At the end of the 1980s, the increasing performance of computers paved the way for the development of optimization techniques aimed at finding the best components values of analog crossover networks [29, 30]. In particular, De

Wit et al. [31] proposed a nonlinear optimization method for designing the crossover filters of a multi-way loudspeaker system. The method is based on a minimization of the difference between the desired and the calculated response of the system in terms of sound pressure and power to achieve optimal values of components such as the power-handling capacity. According to Colloms [1], since the 1980s, the common choice for the crossover point in a two-way speaker system has been about 3 kHz.

Advances in digital signal processing (DSP) hardware technology enabled digital crossover filtering in the late 1980s [32, 33]. Digital filters are attractive because of their easy implementation allowing one to obtain the desired characteristics with the filters placed before the power amplifiers [34]. Aarts and Kaizer [35] proved the feasibility of simulations of both active and passive crossover filters through a digital signal processor. Furthermore, the use of a digital signal processor allows to integrate other functionalities [36], such as equalization for drivers and room compensation. Wilson et al. [32] listed features that digital crossover filters offer that would be impractical using analog techniques, such as linear phase, steep slope, correction of the relative time delay between drivers, adjustment of the overall frequency response using software, and equalization to compensate the response of the drivers for the room in which the system is to be used.

2 PRINCIPLES OF CROSSOVER NETWORK DESIGN

The authors define the basic requirements for the crossover network design for high-quality sound reproduction:

I. Flatness: The on-axis magnitude response of the combined outputs should be flat within ±1.5 dB [19].
II. Steepness: The stop-band cutoff rate of each filter should be steep enough, at least −12 dB/octave [37], to avoid distortion or damaging the drivers.
III. Polar behavior: The polar response of the combined outputs should be uniform and symmetric around $\theta = 0°$ taking into consideration the physical separation of the drivers [33].
IV. Phase linearity: The phase response of the combined outputs should be sufficiently linear around the crossover frequency. A 2-ms tolerance for the group-delay deviation at middle frequencies is approved [38], but at low frequencies, a larger deviation can be acceptable [39].

These requirements are in accordance with some prior works [40, 2]. The first two conditions—flatness and steepness—have been analyzed in detail in the literature [27, 37, 41–43].

The polar response is the magnitude response of the whole crossover network as a function of angle in the free field [32]. It is usually derived from assuming that the ideal listening position is located in the far field, at the same dis-

tance from the drivers, and only at one point because the drivers are not in the same position and are non-coincident. At other points, the listener will be at different distances from each driver, and this leads to flight time differences. These differences in the signals produce attenuation of some frequencies, depending on the listening position.

To face this problem, the crossover network is usually optimized in front of the loudspeaker position, i.e., in the vertical position or in the on-axis response, but not accounting for the loudspeaker directivity characteristics and without studying the behavior outside this position, i.e., attenuation or cancellation in the off-axis response. Because of these aspects, the optimal definition of the crossover polar response is still an open question [44–46]. Furthermore, these aspects are more critical in applications such as Public Address (PA) and sound reinforcement systems, in which the drivers can be far apart, as reported in [47].

For the requirement of an acceptable phase response, the subjective importance of a linear phase is still under debate, and usually, the target is a sufficiently linear phase without affecting the other three properties [32, 48]. Ashley [38, 49] imposed in the 1980s a tolerance of 2 ms for the group delay through the bands of a loudspeaker. It is now known that even smaller group-delay differences of about 0.6 ms may be audible in the middle range [39]. However, at bass frequencies, the hearing is less sensitive in this respect, and thus, larger group-delay tolerances can be used, such as about 3 ms at 500 Hz [39, 50] and about 5 ms at 250 Hz [39, 51].

Another important aspect to take into account for a crossover network design is the frequency response of the involved drivers. In fact, the cutoff frequencies must be chosen according to the working frequency range of each driver. Moreover, the crossover filters can be designed in an efficient way to verify the magnitude flatness of the combined outputs, but the real signal outputs include the response of the drivers that are not perfectly flat. For this reason, equalization techniques may be added to compensate for this effect [52].

Furthermore, the general requirements of implementation costs and latency have to be considered. In the analog crossover networks, the costs depend mainly on the components used, whereas in a digital implementation, the computational complexity must be factored in. The total throughput latency of the sound system should be small enough to allow real-time performance and lip-sync. The maximum allowed latency for the crossover network depends on the use of the loudspeaker and can vary from application to application [28].

2.1 Two-Way Crossover

The crossover design for a two-way system is discussed next. Example magnitude responses of the low-pass and high-pass crossover filter pair, together with their sum, are shown in Fig. 2, in which f_c is the crossover frequency, i.e., the point at which the gains of the low-pass and high-pass filter meet, usually at −6 dB. Assuming ideal loudspeaker units and denoting with $H_L(e^{j\omega})$ and $H_H(e^{j\omega})$ the low-pass
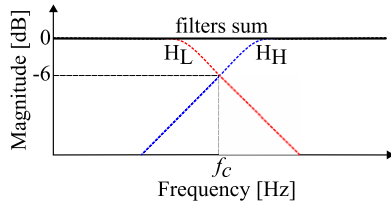
Fig. 2. Magnitude response of a two-way crossover based on Linkwitz-Riley style low-pass and high-pass filters, H_L and H_H.
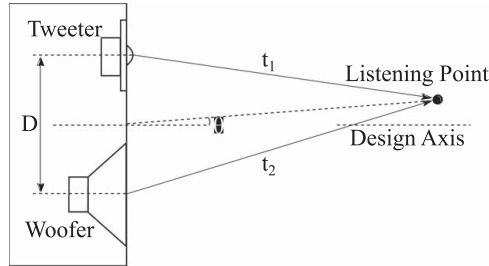

Fig. 3. Two-way loudspeaker system diagram for calculating the polar response, when the driver distance is D and the flight times from the two drivers to the listening point are t_1 and t_2.

and high-pass frequency responses, respectively, the total system response H_T as a function of the vertical listening angle θ (defined in Fig. 3) can be written as

$$H_T(\omega, \theta) = H_L(e^{j\omega}) + H_H(e^{j\omega})e^{-j(t_1 - t_2)}, \qquad (1)$$

where $\omega = 2\pi f$ is the audio frequency (in radians per second), j is the imaginary unit, and t_1 and t_2 are the flight times (acoustic propagation delays) from the tweeter and the woofer, respectively, to the listener point, see Fig. 3.

Requirement I comes true when filters H_L and H_H are all-pass complementary [19, 53] on the design axis ($\theta = 0$):

$$\left| H_L(e^{j\omega}) + H_H(e^{j\omega}) \right| = 1. \qquad (2)$$

After the fulfillment of Eq. (2), requirement II is satisfied if $H_L(e^{j\omega})$ and $H_H(e^{j\omega})$ are also power complementary [54–56], i.e.,

$$\left| H_L(e^{j\omega}) \right|^2 + \left| H_H(e^{j\omega}) \right|^2 = 1; \qquad (3)$$

or magnitude complementary, i.e.,

$$\left| H_L(e^{j\omega}) \right| + \left| H_H(e^{j\omega}) \right| = 1. \qquad (4)$$

This way, it is guaranteed that where $H_L(e^{j\omega})$ has the pass-band, there $H_H(e^{j\omega})$ has the stop-band [54].

Requirement III is achieved when $H_L(e^{j\omega})$ and $H_H(e^{j\omega})$ are in phase around the crossover frequency [19, 57], i.e.,

$$\phi_L(\omega) = \phi_H(\omega), \qquad (5)$$

where $\phi_L(\omega)$ and $\phi_H(\omega)$ are the phase components of $H_L(e^{j\omega})$ and $H_H(e^{j\omega})$, respectively. This property yields a symmetric polar response around the design axis ($\theta = 0$). Finally, for requirement IV, it is requested that the phase response is sufficiently linear around the crossover frequency
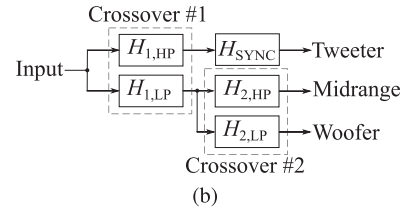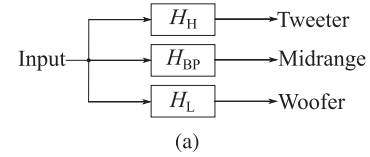

Fig. 4. Three-way crossover network design (a) with three parallel filters [58, 59] and (b) with a tree structure, which requires a phase-synchronization filter H_{SYNC} [60].

so that the group-delay deviation would be smaller than 2 ms in the mid frequencies.

Linkwitz [19] has analyzed the correlation between the different design requirements. He demonstrated the fulfillment of requirement III (symmetric polar behavior) implies that the filters H_L and H_H having the same phase angle at all frequencies, are all-pass complementary [see Eqs. (2) and (5)] and magnitude complementary [Eq. (4)]. However, if the two filters are instead all-pass complementary, according to Eqs. (2) and (5), and power complementary [Eq. (3)], the transfer functions are in the quadrature phase at all frequencies and the peak of the radiation pattern in the vertical plane around the crossover frequency is amplified by 3 dB and tilted toward the lagging loudspeaker driver, so requirement III is not fulfilled [19].

2.2 Three-Way Crossover

Extending to a three-way system and denoting the low-pass, band-pass, and high-pass filters of the crossover with H_L, H_{BP}, and H_H, respectively, similarly to the two-way network, the total response of the system at frequency ω and angle θ can be written as

$$\begin{aligned} H_T(\omega, \theta) = H_L(e^{j\omega}) &+ H_{BP}(e^{j\omega})e^{-j\omega(t_2 - t_3)} \\ &+ H_H(e^{j\omega})e^{-j\omega(t_1 - t_3)}, \end{aligned} \qquad (6)$$

where t_1, t_2, and t_3 are the flight times from each driver to the listener position. The conditions to satisfy requirements I–IV in a three-way system are discussed next.

For the two-way crossover network above, requirement I is achieved by designing two parallel all-pass complementary filters. However, the direct extension of the parallel structure to a three-way crossover network shown in Fig. 4(a) is nontrivial. In particular, although this structure can be realized by combining two-way sections, an all-pass overall response may not be obtained, especially when the crossover frequencies are close to each other [61–63].

To obtain a trouble-free implementation, the three-way crossover network is usually based on a tree structure shown in Fig. 4(b), which is obtained by cascading a pair of two-way crossovers [64, 60]. The same strategy can be applied
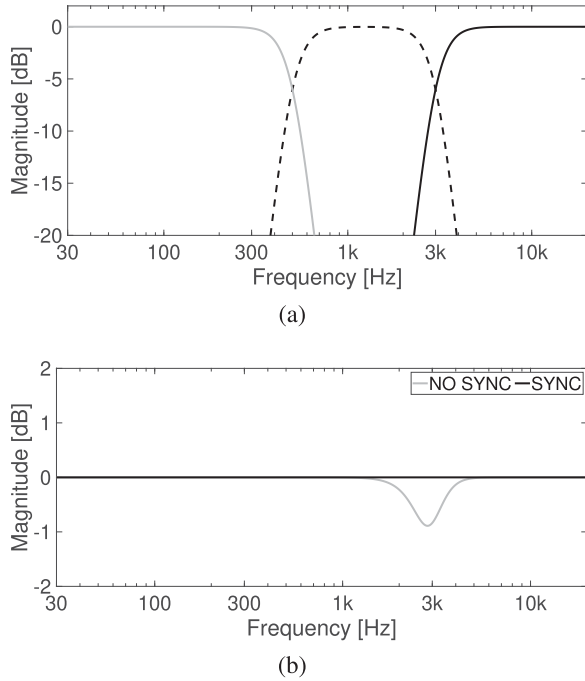
(a)

(b)

Fig. 5. (a) Magnitude responses of woofer (gray line), midrange (dashed line), and tweeter (black line) signal paths of the three-way crossover of Fig. 4(b), and (b) of the total crossover with the synchronization filter and without it.

also to the design of multi-way crossover networks, increasing the number of branches [64].

Referring to Fig. 4(b), the low-pass filter $H_L(e^{j\omega})$ is built as a cascade of two filters, i.e.,

$$H_L(e^{j\omega}) = H_{1,LP}(e^{j\omega})H_{2,LP}(e^{j\omega}), \qquad (7)$$

where $H_{1,\,LP}(e^{j\omega})$ and $H_{2,\,LP}(e^{j\omega})$ are the low-pass filters of the first and second crossover, respectively. Similarly, the band-pass filter $H_{BP}(e^{j\omega})$ is formed as

$$H_{BP}(e^{j\omega}) = H_{1,LP}(e^{j\omega})H_{2,HP}(e^{j\omega}), \qquad (8)$$

where $H_{2,\,HP}(e^{j\omega})$ is the high-pass filter of the second crossover. Lastly, the high-pass filter $H_H(e^{j\omega})$ is defined as

$$H_H(e^{j\omega}) = H_{1,HP}(e^{j\omega})H_{SYNC}(e^{j\omega}), \qquad (9)$$

where $H_{1,\,HP}(e^{j\omega})$ belongs to the first crossover and $H_{SYNC}(e^{j\omega})$ is a synchronization filter shown in Fig. 4(b).

The synchronization filter is inserted in order to compensate for the phase differences between the tweeter and the midrange bands [64, 60]. The filter $H_{SYNC}(e^{j\omega})$ can be designed as an all-pass filter, or, in the simplest case, it can be a copy of the high-pass filter of the second crossover, i.e., $H_{SYNC}(e^{j\omega}) = H_{2,\,HP}(e^{j\omega})$. The absence of the synchronization filter causes a notch around the upper crossover frequency in the on-axis magnitude response of the three-way crossover, as shown in Fig. 5(b).

The three-way crossover of Fig. 5(a) has been obtained using eighth-order Linkwitz-Riley filters [60] with a sampling frequency of 48 kHz. The cutoff frequencies are 500 Hz and 3 kHz, which are typical for three-way loudspeakers

Table 1. Normalized Butterworth polynomials [66].

Order N	Butterworth polynomial $B_N(s)$
1	$s + 1$
2	$s^2 + \sqrt{2}s + 1$
3	$s^3 + 2s^2 + 2s + 1$
4	$s^4 + 2.6131259s^3 + 3.4142136s^2 + 2.6131259s + 1$

[1]. Reviriego et al. have developed similar tree structures for the design of three-way or multi-way crossover networks using IIR all-pass filters [33].

2.3 Crossover Categorization

Crossover filters can be classified into minimum-phase and linear-phase systems. The former is characterized by a minimum phase shift needed to achieve the desired magnitude response, which is the natural behavior of analog filters. Both digital IIR and FIR filters can be used for the implementation of a crossover network. The use of IIR filters aims at a low computational complexity and possibly a minimum-phase behavior, whereas FIR filters are principally used for obtaining linear phase. Many approaches developed in the past for both minimum-phase and linear-phase crossover filter design are reviewed in the next sections.

3 MINIMUM-PHASE CROSSOVER FILTERS

Minimum-phase crossover filters can be categorized into basic approaches that are derived from the analog models, all-pass–based approaches that employ all-pass filtering in all branches of the crossover network, polynomial-based approaches that use well-known polynomial to derive each filter of the crossover, and hybrid approaches that combine IIR and FIR digital filters. Several of these designs were first proposed in the analog world but can be readily converted to digital filters.

3.1 Butterworth Filters

In 1970, Ashley [27, 65, 66] proposed Butterworth filters for crossover network design. These filters were employed in the design of passive crossover networks thanks to the constant input impedance of the combined output property [65]. Butterworth low-pass and high-pass filters of order N are derived from Butterworth polynomial $B_N(s)$, listed in Table 1, as follows [66]:

$$H_L(s) = \frac{1}{B_N(s)} \quad \text{and} \quad H_H(s) = \frac{s^N}{B_N(s)}, \qquad (10)$$

where s is the Laplace variable. The magnitude response of Butterworth filters is maximally flat at 0 Hz, which means that the desired frequency response value and its $N - 1$ derivatives are matched at that frequency [67].

According to requirement II, a first-order filter is not steep enough for a crossover filter, because its response only decays about 6 dB per octave, but second-order and higher ones are, because they decay about $12N$ dB/octave

Table 2. Coefficients of the second and fourth-order digital Butterworth and Linkwitz-Riley crossover filters ($f_c = 3$ kHz, $f_s = 48$ kHz).

Design method	Order N	Digital low-pass filter	Digital high-pass filter
Butterworth	2	$\dfrac{0.029955(1+2z^{-1}+z^{-2})}{1-1.454244z^{-1}+0.574062z^{-2}}$	$\dfrac{0.757076(1-2z^{-1}+z^{-2})}{1-1.454244z^{-1}+0.574062z^{-2}}$
	4	$\dfrac{9.33\times10^{-4}(1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4})}{1-2.976844z^{-1}+3.42231z^{-2}-1.786107z^{-3}+0.355577z^{-4}}$	$\dfrac{0.596302(1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4})}{1-2.976844z^{-1}+3.42231z^{-2}-1.786107z^{-3}+0.355577z^{-4}}$
Linkwitz-Riley	2	$\dfrac{0.027526(1+2z^{-1}+z^{-2})}{1-1.336357z^{-1}+0.446463z^{-2}}$	$\dfrac{0.695705(1-2z^{-1}+z^{-2})}{1-1.336357z^{-1}+0.446463z^{-2}}$
	4	$\dfrac{8.97\times10^{-4}(1+4z^{-1}+6z^{-2}+4z^{-3}+z^{-4})}{1-2.908487z^{-1}+3.262948z^{-2}-1.669652z^{-3}+0.329547z^{-4}}$	$\dfrac{0.573165(1-4z^{-1}+6z^{-2}-4z^{-3}+z^{-4})}{1-2.908487z^{-1}+3.262948z^{-2}-1.669652z^{-3}+0.329547z^{-4}}$

[65]. At the crossover frequency, the magnitude response of a Butterworth filter reaches $1/\sqrt{2}$, or -3.01 dB [27].

In 1975, Thiele [53] showed that odd-order and even-order Butterworth crossover filters behave differently. An even-order Butterworth crossover does not satisfy requirement I, because the magnitude of the combined outputs is not completely flat but shows a 3-dB bump at the crossover frequency. Contrarily, odd-order Butterworth crossover filters can have a flat magnitude response and smooth group delay around the crossover frequency, thus fulfilling requirements I and IV, if they are designed as the cascade of an even-order and a first-order Butterworth filter [53, 19]. However, odd-order Butterworth filters have an asymmetric polar response [19].

As an example, the second-order Butterworth crossover filters are described as follows [66]:

$$H_L(s) = \frac{1}{1+\sqrt{2}s+s^2}, \quad H_H(s) = \frac{s^2}{1+\sqrt{2}s+s^2}. \quad (11)$$

These analog filters can be converted to digital ones using the pre-warped bilinear transform [67, 68]:

$$s = \frac{1}{\zeta}\frac{1-z^{-1}}{1+z^{-1}}, \quad (12)$$

where $\zeta = \tan(\omega_c T/2)$, $\omega_c = 2\pi f_c$ is the crossover frequency in rad/s, $T = 1/f_s$ is the sampling interval at sample rate f_s, and z is a complex variable used in the z-transform.

Substituting Eq. (12) to Eqs. (11) yields the following digital IIR filter transfer functions in the z domain:

$$
\begin{aligned}
H_L(z) &= \frac{g_L(1+z^{-1})^2}{1+a_1z^{-1}+a_2z^{-2}}, \\
H_H(z) &= \frac{g_H(1-z^{-1})^2}{1+a_1z^{-1}+a_2z^{-2}},
\end{aligned} \quad (13)
$$

where the gains g_L and g_H are calculated as

$$g_L = \frac{\zeta^2}{1+\sqrt{2}\zeta+\zeta^2} \ , \ g_H = \frac{1}{1+\sqrt{2}\zeta+\zeta^2}, \quad (14)$$

and the coefficients a_1 and a_2 are

$$a_1 = -\frac{2(1-\zeta^2)}{1+\sqrt{2}\zeta+\zeta^2} \ , \ a_2 = \frac{1-\sqrt{2}\zeta+\zeta^2}{1+\sqrt{2}\zeta+\zeta^2}. \quad (15)$$

Examples of digital Butterworth filters with $N = 2$ and $N = 4$ are included in Table 2, where the crossover frequency is $f_c = 3$ kHz and the sample rate is $f_s = 48$ kHz.
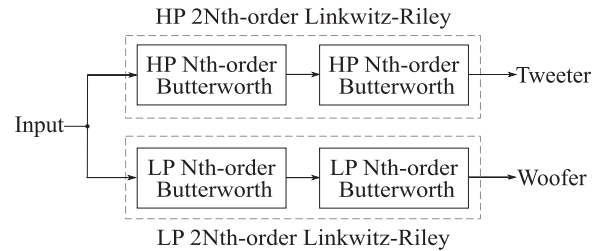

Fig. 6. Linkwitz-Riley crossover network of order $2N$, derived by the cascade of two identical Nth-order Butterworth low-pass (LP) and high-pass (HP) filters.

3.2 Linkwitz-Riley Filters

In 1976, Linkwitz proposed to cascade two identical Butterworth filters to form even-order low-pass and high-pass filters for the crossover network [19]. He credited Riley for having suggested this to him [19], and for this reason, the structure became known as the Linkwitz-Riley crossover , although it is sometimes called the squared Butterworth network. Good properties of the Linkwitz-Riley crossover include a flat on-axis magnitude response (fulfilling requirement I) and minimal phase difference between the low-pass and high-pass filter, although it is not a linear-phase system (still fulfilling requirement IV). Consequently, Linkwitz and Riley's solution became very popular in loudspeakers and other audio applications [40, 69–71]. Fig. 6 shows the block diagram for the design of a $2N$th-order Linkwitz-Riley two-way crossover network, realized using pairs of Nth-order Butterworth filters.

For example, the second-order Linkwitz-Riley crossover network is built using the following squared (i.e., cascaded) first-order Butterworth low-pass and high-pass filters [19]:

$$H_L(s) = \left(\frac{1}{1+s}\right)^2 \text{ and } H_H(s) = \left(\frac{s}{1+s}\right)^2. \quad (16)$$

Substituting Eqs. (12) to (16) leads to the following digital IIR filter transfer functions:

$$H_L(z) = \frac{g_L(1+z^{-1})^2}{(1+a_1z^{-1})^2}, \quad H_H(z) = \frac{g_H(1-z^{-1})^2}{(1+a_1z^{-1})^2}, \quad (17)$$

where the gains are $g_L = \zeta^2/(1+\zeta)^2$ and $g_H = 1/(1+\zeta)^2$, and the feedback coefficient is
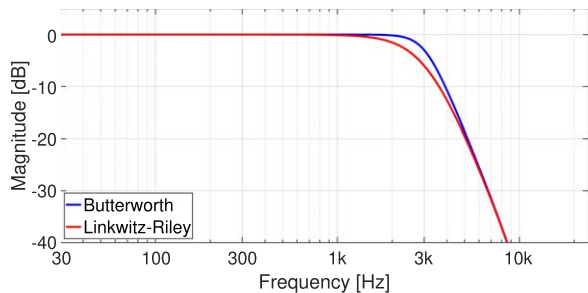
$$a_1 = -\frac{1-\zeta}{1+\zeta}. \quad (18)$$

Fig. 7. Comparison of fourth-order Butterworth and Linkwitz-Riley digital low-pass filters with the cutoff point at $f_c = 3$ kHz.
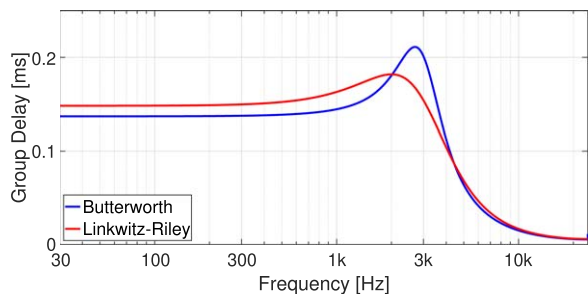

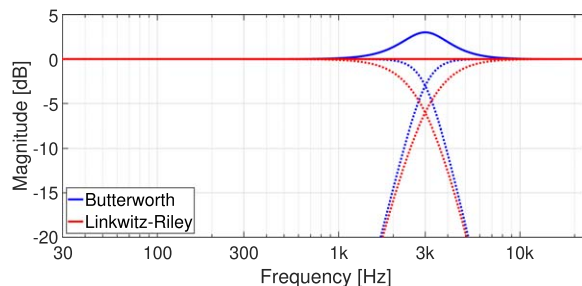Fig. 8 Comparison of group delays of the filters of Fig. 7.

Fig. 9. Magnitude responses of fourth-order Butterworth and Linkwitz-Riley two-way crossovers with a cutoff frequency of $f_c = 3$ kHz. The dashed lines correspond to the low-pass and high-pass filters, and the solid lines are the total responses.
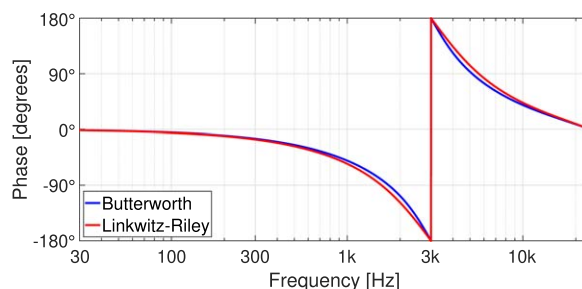

Fig. 10 Phase responses of the crossovers of Fig. 9.

Examples of the z-domain transfer functions of these digital Linkwitz-Riley low-pass and high-pass filters, as well as the fourth-order ones, with the crossover point at $f_c = 3$ kHz, are shown in Table 2.

Because Eq. (17) shares the same denominator, and the numerators have only one non-trivial coefficient, as can also be seen in Table 2, the second-order Linkwitz-Riley filters are more economical to implement than two regular biquad sections, which is another charming feature of this popular structure. The Linkwitz-Riley crossover network can be derived from any order of Butterworth filters, and can alternatively be realized with digital all-pass filters [71].

A comparison between the digital Butterworth and Linkwitz-Riley filters presented in Table 2 has been carried out. Fig. 7 shows the fourth-order low-pass filter magnitude responses, and Fig. 8 shows their group delays. The magnitude responses of the filters are similar, but have a different gain at the cutoff frequency in Fig. 7, -3 and -6 dB. Although the phase response of these IIR methods is not completely linear, their group delay varies much less than the limit of 2 ms in Fig. 8, so both filters satisfy requirement IV. The group-delay deviation of the Linkwitz-Riley filter is slightly smaller than that of the Butterworth filter due to the lower quality factor achieved by the cascading of the two identical second-order Butterworth filters.

Figs. 9 and 10 show the magnitude and phase response of the Butterworth and Linkwitz-Riley filters, taking into consideration the sum of a two-way crossover. The difference is that the Butterworth crossover has a 3-dB boost at the crossover point, whereas the Linkwitz-Riley crossover is flat. Therefore, the Linkwitz-Riley crossover satisfies requirement I but the Butterworth crossover does not.

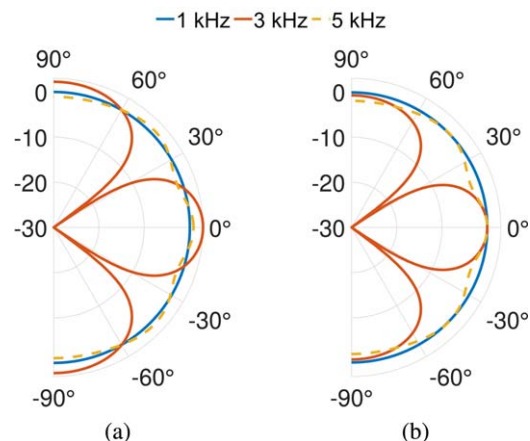Fig. 11 shows the polar response of the fourth-order

Fig. 11. Polar plot of fourth-order IIR crossover network using the method of (a) Butterworth and (b) Linkwitz-Riley, with a cutoff frequency of 3 kHz, distance between loudspeakers of $D = 0.1$ m and distance from the origin $R = 1$ m.

crossover filters at three different frequencies, and Fig. 12 shows the total magnitude response at four different vertical angles. The figures are obtained following the scheme of Fig. 3 and through the implementation of Eq. (1) with $R = 1$ m, $D = 0.1$ m, and $f_c = 3$ kHz. Both methods show a uniform and symmetric distribution, so requirement III is verified by both Butterworth and Linkwitz-Riley crossover networks. Nonetheless, the 3-dB bump at the crossover frequency of the Butterworth method is evident also in the polar response.
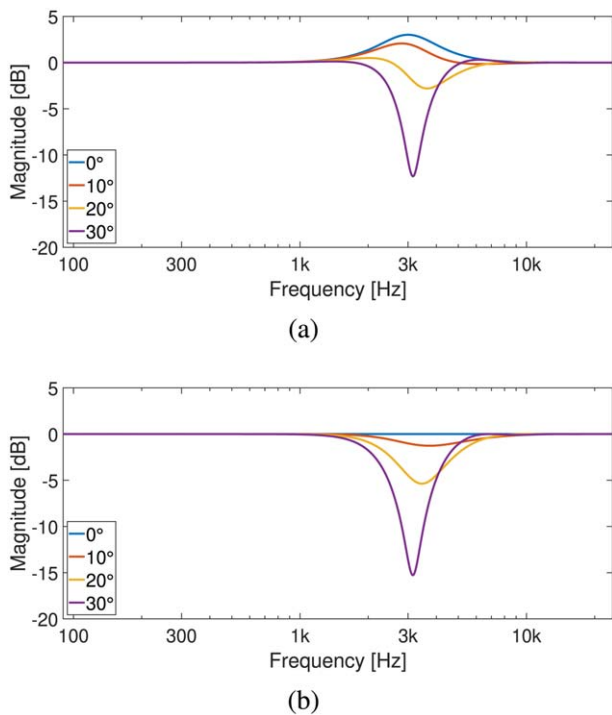
Fig. 12. Magnitude responses of the (a) Butterworth and (b) Linkwitz-Riley filters of Fig. 11 at different vertical angles.
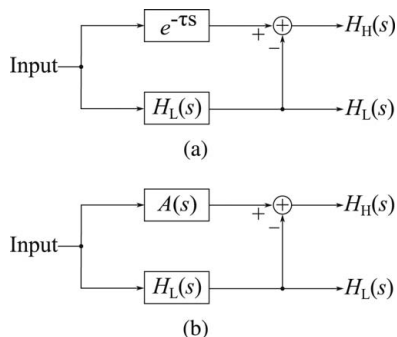

Fig. 13. (a) Time delay–derived subtractive [40] and (b) in-phase subtractive crossover network structures presented by Lipshitz and Vanderkooy [73].

3.3 Other Analog Models

Several structures for the implementation of Linkwitz-Riley [19] crossover networks were presented over the years to improve their performance. In 1985, Chalupa reported a practical active implementation for second-order and fourth-order Linkwitz-Riley structure based on a subtractive approach in which the high-pass response is obtained by subtracting the low-pass response from the input [69]. A similar approach has been proposed in [70, 72].

In 1983, Lipshitz and Vanderkooy [40] noted that the subtractive approach [69] generated a complementary high-pass filter with a low roll-off rate. This was improved by adding a time delay in the high-pass band. In this way, a new class of linear-phase and high-slope crossover filters was proposed. The structure of [40] is shown in Fig. 13(a), where H_L and H_H are the final transfer functions of the low-frequency and high-frequency bands, respectively, and

τ is the time delay chosen to equal the 0-Hz phase and group delays of the low-pass filter. Afterwards, Lipshitz and Vanderkooy developed their structure further by replacing the delay with an all-pass filter, as shown in Fig. 13(b) [73]. Rapoport et al. studied a similar structure later [74]. Other crossover networks based on all-pass filters are discussed in the next subsection.

In 2007, Thiele [75] proposed an asymmetric filter structure based on the Linkwitz-Riley approach, demonstrating that low-pass and high-pass filters can have different orders because of the different characteristics of the two drivers. This aspect has also been analyzed and confirmed by Hawksford [76], showing the importance of having an efficient crossover structure.

3.4 All-Pass–Based Approaches

The possibility of achieving a quasi-linear phase crossover network using all-pass filters has been analyzed by several researchers, showing that it is possible only at the expense of polar behavior [40, 77–79]. As Lipshitz and Vanderkooy have reported [73], the optimal polar behavior can be obtained by developing high-pass and low-pass outputs that are in phase through the crossover region, but it is derived at the expense of the phase response that becomes non-linear. Following this idea, Lipshitz and Vanderkooy proposed a modified structure shown in Fig. 13(b), which consists of substituting the pure time delay with an all-pass filter [40]. The Linkwitz-Riley implementation results as a particular case of this class of in-phase crossovers. Fig. 14 reports the magnitude, phase response, and group delay of the two structures of Fig. 13 showing the fulfillment of requirement I and IV as described in [73].

In 1985, Kyono and Fujiwara proposed a four-way crossover system guaranteeing the in-phase and linear-phase requirements at crossover frequencies [77]. Starting from the time delay derived crossover of [80], the system introduces low-pass and all-pass Bessel filters for the network implementation satisfying requirements I, III, and IV. After some experiments, Kyono and Fujiwara underlined the potential of the approach in the digital domain and the need of improving phase linearity operating on the distortions introduced by the loudspeaker's driver [77].

In the 1980s, D'Appolito [64, 81] presented an extension of the Linkwitz-Riley crossovers to a multi-way crossover network using all-pass filtering. In particular, the extension allows building a multi-way all-pass crossover by exploiting a binary tree structure based on a two-way crossover built with Butterworth polynomials and synchronization filters as phase compensating functions. Moreover, under certain conditions, the crossover can be further simplified by removing synchronization filters and a subtractive structure can be used to reduce the filtering operations, offering a valid solution with low computational complexity.

In 1998, an approximately linear-phase IIR crossover design based on all-pass filters was presented by Reviriego et al. [33]. Starting from the fact that any filter can be decomposed into two all-pass filters [82], the sum and difference of two all-pass filters (one of which is a delay line) are used
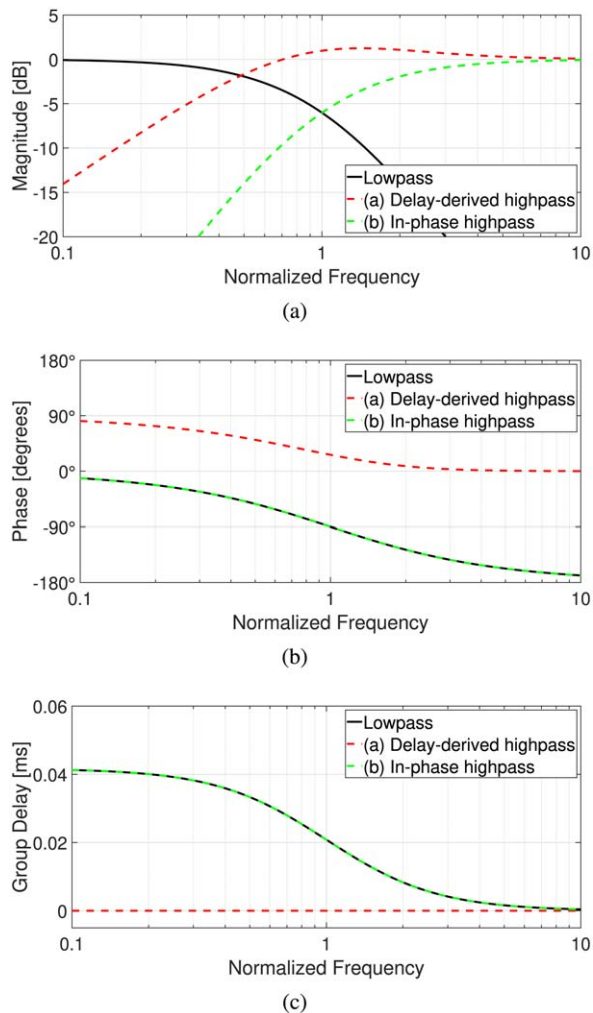
(a)

(b)

(c)

Fig. 14. Comparison between the time delay-derived crossover of Fig. 13(a) and the in-phase crossover of Fig. 13(b), in terms of (a) magnitude response, (b) phase response, and (c) group delay, using a second-order Linkwitz-Riley low-pass filter.
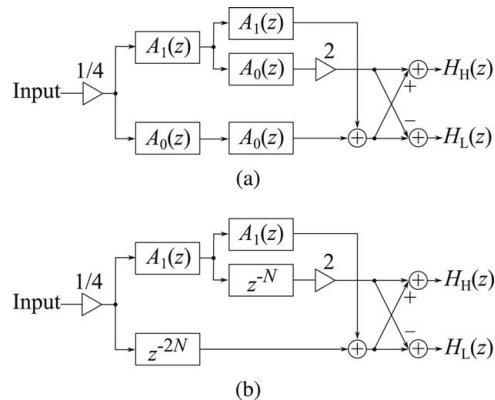

(a)

(b)

Fig. 15. Efficient implementation of a two-way crossover proposed by Reviriego et al. [33] based on all-pass filtering, with (a) Linkwitz-Riley filters and (b) in-phase filters.
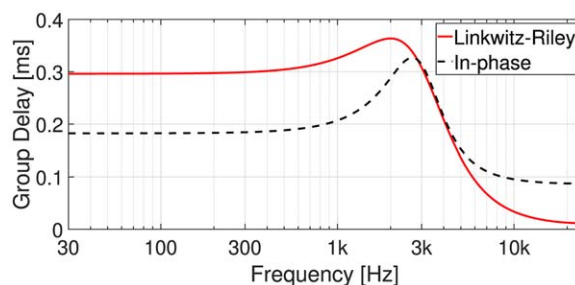

Fig. 16. Group delay comparison between an order eighth Linkwitz-Riley design and an order 10 in-phase crossover of Reviriego et al. [33], when the sample rate is 48 kHz.

to build a crossover network. Fig. 15(a) shows the application of this concept to the Linkwitz-Riley approach, and Fig. 15(b) shows the approach proposed by Reviriego et al. In particular, $A_0(z)$ and $A_1(z)$ represent the all-pass decompositions, and in the proposed approach, it is imposed that $A_0(z) = z^{-N}$. A Butterworth, Chebyshev, or elliptic low-pass filter $G(z)$ may be used for the decomposition, i.e., $G(z) = [A_0(z) + A_1(z)]/2$ [52]. Reviriego's method outperforms the original Linkwitz-Riley filters in terms of complexity and peak group-delay deviation, as shown in Fig. 16, at the cost of a wider transition band.

Reviriego et al. also generalized their in-phase design for a multi-way structure [33]. As an example, a three-way implementation derived from the design of two pairs of in-phase crossover filters showed great performance in terms of computational complexity with the disadvantage of some phase distortion [33]. Nevertheless, when the final phase distortion is still present because of the all-pass structure, it is possible to apply an equalization procedure to overcome this problem [83, 84].

Sookcharoenphol et al. [85, 86] proposed, starting from a two-way crossover realized with parallel all-pass filters, a real-time implementation based on a class of linear-phase IIR filters [87]. These filters, which are derived from a time-reversal section and a truncated impulse response technique, exhibit a flat group delay, i.e., a linear phase, and steep slopes. In the proposed work these filters are derived from an elliptic prototype and the two-way crossover is compared with a fourth and eighth-order Linkwitz-Riley crossover. Simulated results [86] show that the proposed network has a flat magnitude response and an approximately linear phase with a steep cutoff rate and low computational cost, satisfying requirements I, II, and IV.

3.5 Polynomial Approaches

Several works have considered Bessel polynomials to form a crossover network [88, 89, 77, 90]. However, traditional Bessel filters [89] are unsuitable for a crossover network, because the high-pass filter does not have a linear phase, attenuation occurs at the crossover frequency, and the summed response is not flat, so requirements I and IV are not verified. Miller showed that with some constraints it is possible to derive a high-pass filter from its low-pass counterpart with an almost linear phase and a flat magnitude response [90].

In 2010, Zhang et al. presented a method for the design of digital crossovers based on Bessel filters [91]. Start-

ing from Miller's work [90], the proposed methodology is aimed to give a procedure to obtain digital coefficients for a DSP implementation. In particular, the low-pass and high-pass analog filters are derived from the normalized Bessel polynomial functions; then, the normalization is removed; and finally, the digital filter is obtained with the bilinear transform. The resulting digital Bessel filters can be implemented with first and second-order sections [91].

The implementation of a three-way crossover based on Bernstein polynomials was proposed by Chutchavong et al. [58, 59]. One of the most important characteristics of filters derived from Bernstein polynomials is that they exhibit a maximally flat magnitude both on band-pass and stop-band and almost linear phase. Moreover, it is possible to set the stop-band attenuation and the phase slope with some polynomial parameters that allow deriving the filters coefficients. The filters are obtained in the analog domain and are then converted into digital filters by using the bilinear transformation. The experimental results show good performance in terms of flat magnitude and computational efficiency. Also Hlurprasert et al. [92] proposed an approach based on Bernstein polynomials to develop a four-way crossover network. They report results similar to [58, 59], showing good performance obtaining a flat magnitude response and a zero-phase system.

In 2013, Huber [93] proposed a family of all-pole filters based on Gengebauer polynomials suggesting that they can be used for the design of crossover networks. The peculiarity is that they can match the specifications of Butterworth filters using a lower order [93] and thus provide a useful alternative.

3.6 Hybrid Digital Crossover Design

To overcome the disadvantages of IIR and FIR implementations and to exploit their strengths, i.e., computational load and phase linearity, respectively, mixed solutions for crossover filters design have been proposed [94, 95, 28]. The idea of mixing FIR and IIR structures allows obtaining a structure with a reduced delay but with minimum-phase characteristics and it has been successfully introduced in other applications such as adaptive filtering [96], digital reverberation [97], and filter banks [98].

The idea of the hybrid digital crossover network presented by Palestini et al. [95] is to realize a number of low-frequency bands using IIR filters to have a reduced delay, and to implement the high-frequency bands by means of FIR filtering keeping a linear phase response in the upper part of the spectrum. This leads to a tree structure, with the FIR filter tree grafted on the upmost branch of the IIR one, as seen in Fig. 17. The IIR tree structure is subtractive: even though this scheme does not lead to a minimum-length graph, it is extensible to any number of bands. Each element in Fig. 17 is realized as follows:

$$ G(z) = \left[\frac{z^{-N} + A(z)}{2} \right]^2 \qquad (19) $$
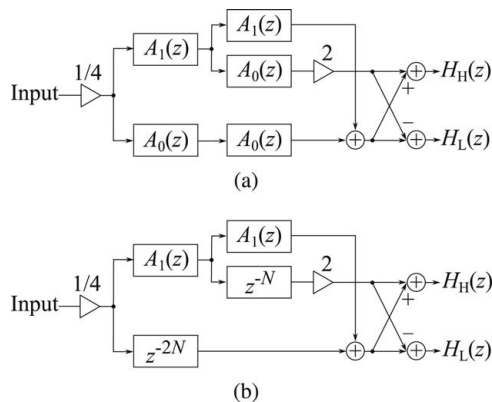
and

$$ D(z) = z^{-N} A(z), \qquad (20) $$

Fig. 17. Hybrid digital crossover network proposed by Palestini et al. [95] in which the low-frequency bands are implemented as IIR filters, whereas high-frequency bands have also an FIR part.

Table 3. Comparison between different four-way crossovers whose filters have approximately the same stop-band attenuation (TB is the transition band) and a sample rate of 44.1 kHz as reported by Palestini et al. for the hybrid crossover network [95].

Type	Filter TB (Hz)	Order	Latency (ms)	Phase
FIR	60–100	5,000		Linear-phase
	250–350	1,700	81	
	2,800–3,200	400		
IIR	60–100	559		Approximately
	250–350	259	21	linear except
	2,800–3,200	99		in TBs
Hybrid	60–100	559		Approximately
	250–350	1,700	37	linear except in
	2,800–3,200	400		IIR TBs

where $G(z)$ is a low-pass IIR filter and $D(z)$ its respective all-pass phase term needed to properly align the signals of different bands, and N is the order of the all-pass filter $A(z)$ that is designed using a weighted least squares equation error approach [99] to meet a linear phase specification.

Similarly, the FIR tree scheme has also been implemented in a subtractive way as depicted in Fig. 17. Starting from an even order N symmetric low-pass–filter $H_L(z)$ designed by weighted constrained least square [100, 101], its corresponding high-pass filter $H_H(z)$ is derived in the time domain as follows:

$$ H_H(z) = z^{-N/2} - H_L(z). \qquad (21) $$

Table 3 shows the performance of the hybrid structure in comparison with an FIR implementation and IIR implementation. The advantages of the hybrid structure are evident: the structure obtained all the positive aspects of the IIR and FIR, i.e., reduced computational complexity keeping a quasi-linear phase response.

Fig. 18 shows the behavior of a four-way crossover obtained with the structure of Fig. 17 containing one FIR and two IIR filters. Analyzing the group delay response of Fig. 18(b), the influence of IIR filters at low frequencies is evident from the group-delay peak, which also marginally
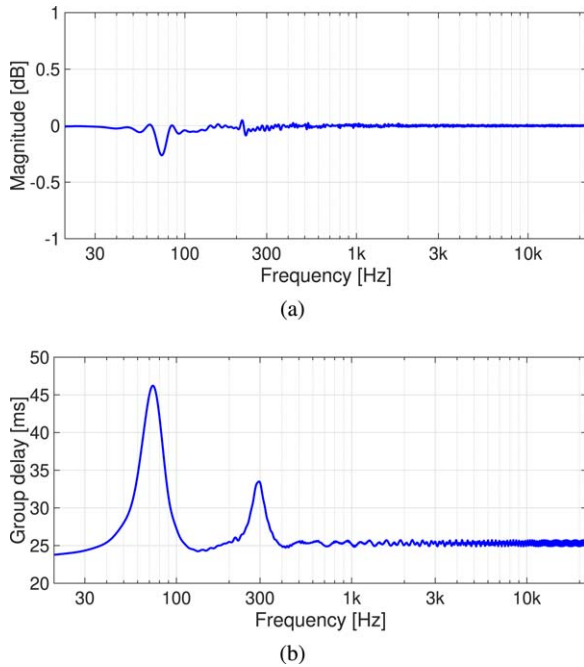
(a)

(b)

Fig. 18. (a) Magnitude response and (b) group delay of four-way total hybrid (IIR/FIR) crossover network [95].

affects the magnitude response in Fig. 18(a). However, the group-delay oscillation widely exceeds the 3-ms and 5-ms limits at low frequencies, avoiding the fulfillment of requirement IV.

Di Cola et al. [28] presented a practical implementation of a mixed IIR-FIR crossover network. The proposed structure does not provide a perfect linear-phase system but something that is very similar to a minimum-phase system.

3.7 Other Minimum-Phase Approaches

Thiele [102] presented a class of crossover systems that produces a null response in their high-pass and low-pass outputs at frequencies closer to the transition frequency. This allows the outputs to have a high initial rate of attenuation in the stop-bands and a flat all-pass response when filters are summed, thanks to the fact that high-pass and low-pass sections are exactly in-phase [103].

Focusing on the off-axis polar response, Catalá [47] proposed an approach derived from the traditional Linkwitz-Riley crossover. In particular, the main idea is to involve the measurement of a complex 3D response of the loudspeaker with the method proposed by Feistel et al. [104] to estimate the best crossover for each angle. Then, a correction filter based on a listening window is used to derive the final crossover characteristic. The approach allows to improve the off-axis performance of the speaker near the crossover frequency, and this is particularly important, especially with PA systems. On the other side, the method could degrade the performance of on-axis listening because the crossover also takes into consideration the off-axis behavior and the final response is a sort of compromise.
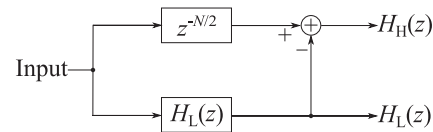

Fig. 19. Linear-phase FIR crossover implementation employing a complementary structure [32].

4 LINEAR-PHASE CROSSOVER FILTERS

Linear-phase crossover networks can be categorized in time-domain approaches, multi-rate approaches, magnitude response approaches, and other approaches based on more special techniques, such as the vector space projections method, or B-spline functions. In the following, each category is described and analyzed.

4.1 Time-Domain Models

A straightforward way of designing a linear-phase crossover filter is to cascade an IIR Butterworth filter with its time-reversed version. The time-reversed filter can be implemented as an FIR filter, which samples the truncated Butterworth filter impulse response. However, this derivation produces a filter with a very long latency, if the truncation error is kept minimal. Fig. 20 shows an example in which a linear-phase FIR filter designed this way is compared with a Linkwitz-Riley filter. To overcome this problem, some approaches were proposed in the literature.

In an early approach to FIR crossover network design, Wilson et al. [32] show a real-time implementation of a digital crossover network based on FIR filters and subtractive structure as shown in Fig. 19, following Eq. (21) with $z = e^{j\omega}$. The entire system of Fig. 19 can be synthesized as follows:

$$
\begin{aligned}
&H_{\mathrm{L}}(e^{j\omega}) + H_{\mathrm{H}}(e^{j\omega}) \\
&= \left| H_{\mathrm{L}}(e^{j\omega}) \right| e^{-jk\omega} + \left| H_{\mathrm{H}}(e^{j\omega}) \right| e^{-jk\omega} \\
&= \left| H_{\mathrm{L}}(e^{j\omega}) + H_{\mathrm{H}}(e^{j\omega}) \right| e^{-jk\omega} = e^{-jk\omega}.
\end{aligned} \tag{22}
$$

In this way, the design completely satisfies the property of a flat magnitude response, a steep cutoff of the individual filters, and a linear phase response of the combined output. At the same time, a near-ideal polar response, as with FIR filters, and a very steep cutoff frequency can be obtained. An ideal polar response of the combined output requires infinite cutoff rate filters, in principle [32]. Around the same time, the same structure was independently proposed by Schuck and Klowak [105].

Fig. 20 compares Wilson's FIR crossover design of [32] with the linear-phase time-reversed Linkwitz-Riley FIR filter and the minimum-phase Linkwitz-Riley IIR filters [19], considering a cutoff frequency $f_{\mathrm{c}} = 3$ kHz. In particular, the Wilson's FIR filter is designed with the Parks-McClellan algorithm having a stop-band attenuation of -80 dB, a pass-band ripple of 0.1 dB, and a transition bandwidth of 3 kHz to produce a slope similar to that of the IIR filter, as shown in Fig. 20(a). The Wilson's FIR filter order is $N_{\mathrm{F}} = 18$, whereas the IIR filter order is $N_{\mathrm{I}} = 4$, and the
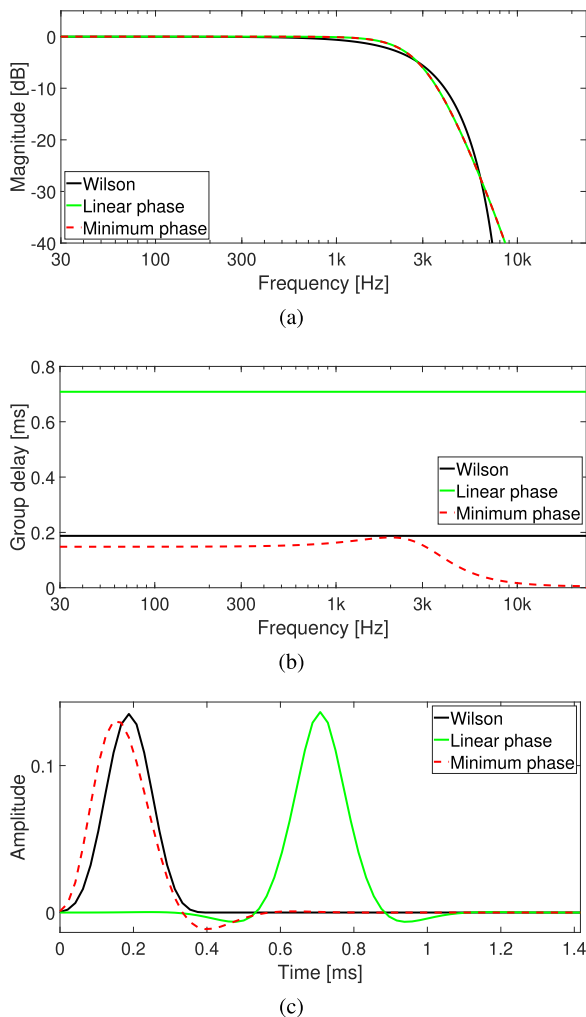
(a)

(b)

(c)

Fig. 20. Comparison of (a) magnitude responses, (b) group delays, and (c) impulse responses of the linear-phase FIR filter of Wilson [32], the linear-phase time-reversed Linkwitz-Riley FIR filter, and minimum-phase IIR fourth-order Linkwitz-Riley low-pass filters with a cutoff at 3 kHz.
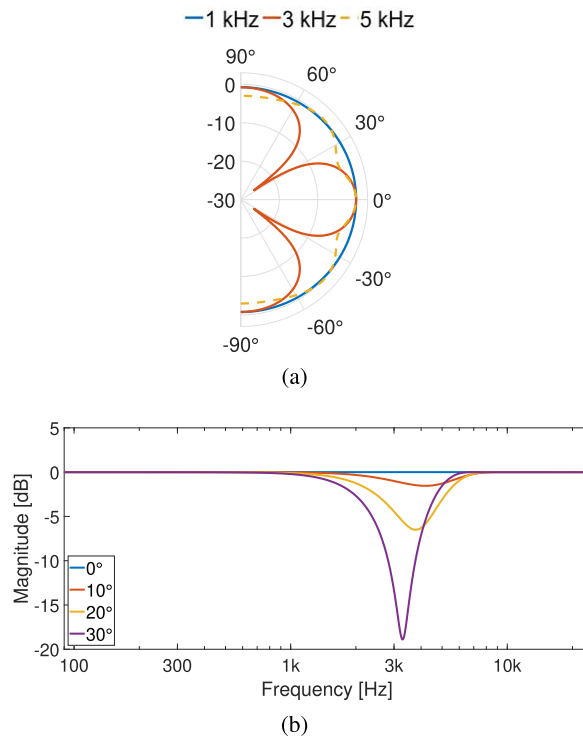

(a)

(b)

Fig. 21. Linear-phase FIR crossover network of order $N_F = 18$ with a crossover frequency of 3 kHz: (a) polar plot at three frequencies and (b) the total magnitude response at different angles.

sample rate is 48 kHz. The time-reversed IIR Butterworth impulse response is truncated at 35 samples, resulting in a final linear-phase filter of order 68.

In Fig 20(b), it is seen that the FIR filters have a constant group delay, i.e., a linear phase response. Although the group delay introduced by the Wilson's FIR filter is slightly larger than that of the IIR filter in 20(b), both are very small, less than 0.2 ms, because the cutoff frequency is quite high. On the contrary, the group delay of the linear-phase time-reversed Linkwitz-Riley FIR filter is bigger than Wilson's FIR filter due to its derivation. Fig. 20(c) shows that the FIR filter's impulse response is symmetric, whereas the IIR filter's response is non-symmetric, but the delay is smaller. Moreover, a potential disadvantage of the FIR approach is the slow rise of its impulse response in Fig. 20(c), which may cause an undesired pre-ringing effect, when the FIR filter has a strict specification and very high order [106].

Fig. 21 shows the polar response of the crossover at three different frequencies and the total magnitude response at four listening angles in the listening window. The graphs of

Fig. 21 are obtained following Eq. (1), imposing a listening distance of $R = 1$ m and a driver distance of $D = 0.1$ m, cf. Fig. 3. In comparison with the responses of the IIR crossover networks of Figs. 11 and 12, the linear-phase FIR crossover shows a behavior similar to the Linkwitz-Riley crossover network [19], confirming the validity of the FIR approaches also in terms of polar response.

4.2 Multi-Rate Approaches

Hämäläinen proposed an FIR digital crossover network based on a multi-rate structure [107]. His approach exploits a multi-rate complementary structure that can be optimized using a suitable criterion, such as the minimal run time memory or the computational complexity. In particular, the structure is realized as a cascade of decimation stages, filters, and interpolation stages, and its optimization procedure is based on the frequency sampling method in the weighted least mean squared sense.

4.3 Frequency-Domain Approaches

Frequency-domain approaches for crossover network design can solve problems that may occur in time-domain techniques. In fact, the time-domain realizations by Wilson et al. [32] and Schuck [108] offer implementation simplicity but have problems in reaching steep transition bands (requirement II) with limited frequency resolution when low-order filters are used. To ease this problem, Azizi et al. [109] proposed a Fourier transform–based approach. In particular, the desired low-pass filter of the crossover is designed by exploiting the frequency sampling algorithm

[67]. Then, the complementary high-pass filter can be implemented with a subtractive method, as shown in Fig. 19. Finally, the overlap-and-add method is used for real-time applications, obtaining computational complexity savings [109] at the expense of latency.

Another issue that arises with time-domain approaches [32, 108], is the fact that high cutoff rates are obtained with long crossover filters, which may cause undesired pre-ringing effects and large computational burden. These aspects were studied by Greenfield [110], who proposed the pseudo-analog filter alignment to avoid these problems. This method consists of designing the magnitude response of an analog FIR filter with more relaxed specifications than brick-wall filters and of approximating it with the sampling frequency approach.

In the late 1990s, Hawksford [111, 112] presented a generalized FIR filter design technique that matches the magnitude response of analog low-pass and high-pass filters. Zero-phase Butterworth filter prototypes are applied using a raised-cosine weighting function and overlapping filter responses. This method provides a large attenuation in the stop-band with the possibility of choosing arbitrarily the value of the filter slope. Furthermore, two approaches for reducing the polar response error due to the time delay introduced by non-coincident drivers were discussed [112].

Another linear-phase crossover network for multi-way loudspeakers was introduced by Horbach [113]. The proposed technique is based on the definition of a crossover frequency response shape that guarantees a flat magnitude response at a specified vertical off-axis angle. The method has been compared with prior IIR crossover techniques, showing how the proposed crossover maintains a flat off-axis magnitude response throughout most of the operating band of the loudspeaker, except at high frequencies [113].

4.4 Other Linear-Phase Designs

In 1999, Haddad et al. proposed a technique for designing digital linear-phase FIR crossover systems based on the principle of vector space projections method (VSPM) [114]. VSPM is an iterative algorithm yielding FIR filters that verify a given set of imposed constraints, such as the cutoff frequencies of the crossover filters, the number of bands, and a flat magnitude over the entire frequency spectrum. This method could be extended to a multi-way solution and was tested in a three-way crossover system, proving a relatively limited computational cost [114].

In 2019, Stanciu et al. proposed the use of the B-spline function for the design of multi-way crossover network [115]. The crossover filters are derived from an FIR low-pass prototype designed in the frequency domain with the introduction of the B-spline function to approximate the transition bandwidth. The obtained crossover satisfies the above-mentioned requirements with the advantage of a reduced computational complexity.

Recently, Bruschi et al. proposed a linear-phase crossover network design based on interpolated FIR (IFIR) filters, which is implemented as a cascade subtractive structure [116]. The IFIR filters achieve strict specifications, such as
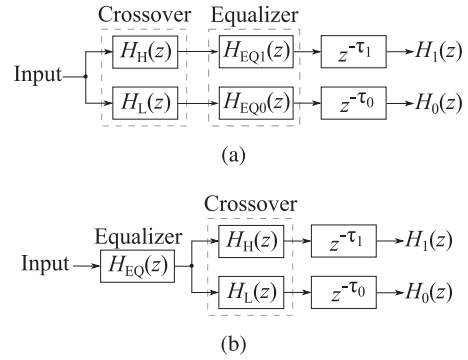

Fig. 22. (a) Band-specific equalizers and (b) a single equalizer applied before the crossover network in a two-way system.

a narrow transition band, with a low computational complexity, guaranteeing a linear phase. Experimental results showed that the IFIR crossover network verifies all the requirements and exhibits a low computational cost in comparison with other FIR crossover techniques [116].

5 EQUALIZATION AND CROSSOVER FILTERS

A loudspeaker may introduce artifacts on the reproduced sound because of the non-ideal frequency response of its drivers. In fact, the ideal frequency response of the reproducing system should be flat for all frequencies, but this rarely happens in reality. For this reason, equalization is often applied to improve loudspeakers. There are two different principles: either every band is equalized individually by combining an equalizer with each crossover filter, as shown in Fig. 22(a), or the input of the loudspeaker is pre-processed with a single equalizer before the signal is split into different bands, see Fig. 22(b). This section analyzes the two different principles of equalization focusing on loudspeaker correction and avoiding the room compensation problem. A detailed review on room equalization is available elsewhere [117].

5.1 Per-Band Equalization

Starting from the idea of compensating for the loudspeakers' non-ideal response, several approaches focus on the band responses measurement and their equalization [Fig. 22(a)]. As presented in the holistic analysis of Hawksford, loudspeaker equalization and crossover network design can be performed in time and frequency domain [111]. Wilson et al. [32] proposed an equalization filter derived in the frequency domain as the inverse function of the measured response. A similar procedure was applied by Schuck and Klowak [105, 108], in which the response inversion was performed exploiting a complex least square approximation technique and decimation is applied to reduce the number of coefficients in the low-pass equalizing filter, as reported in Fig. 23.

Kyono et al. [118, 119] proposed an off-line adaptive procedure to derive a linear-phase equalizer in the time domain. Azizi et al. [109] applied the frequency sampling approach to the inverse of the magnitude responses of the
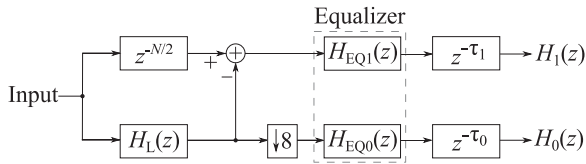
Fig. 23. Two-way multi-rate digital crossover network [105] in which τ_0 and τ_1 are the synchronization delays.
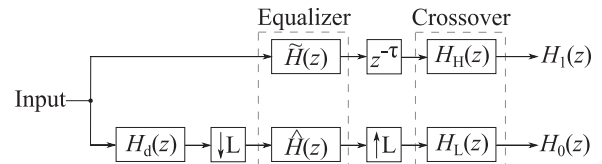

Fig. 24. Multi-rate system with delay compensation and a separate equalizer for each driver [122].

drivers in their pass-bands, and they cascaded those sampled responses with the crossover network.

Focusing also on the idea that the equalization procedure can be developed as part of the crossover design, Reviriego et al. [120] propose the compensation of the non-ideal phase response of the loudspeakers by combining it with the Linkwitz-Riley design and taking into account the listeners placed off-axis. The work showed that it was possible to compensate the off-axis behavior with all-pass filters and could be integrated within the crossover filter adding a proper project constraint [120].

Also the VSPM approach proposed by Haddad et al. [114] includes the equalization in the crossover network, by adding a constraint on the compensation of the loudspeaker frequency responses. Experimental results showing a peak-to-peak deviation of 0.1 dB in the equalized magnitude response were shown [114].

Thaden et al. [94] proposed an equalization method based on the inversion of the measured frequency response of the individual transducers. Each inverted frequency response is multiplied by the required crossover filter response, such as that of a Linkwitz-Riley filter, creating an ideal filter. The relative impulse response is obtained using the inverse fast Fourier transform, and a windowing method allows for the reduction of the impulse-response length [94]. Depending on the design of the impulse response, an FIR filter with a perfectly linear phase response, with a matching latency, or a minimum phase response, with a smaller latency, can be realized [94].

Another approach for loudspeaker compensation combined with crossover design has been suggested by Ramos and Tomas [121], who used a cascade of second-order IIR filters. To improve the method, the possibility of automatically inserting the equalization procedure to the low-pass and high-pass filters was introduced, to enhance the crossover design stage and to obtain precise alignments with lower-order filters.

In 2014, Shavelis et al. [122] proposed an efficient approach to insert equalization filters to a crossover network with a low computational complexity, exploiting decimation. Fig. 24 shows the overall scheme, in which $\hat{H}(z)$ is the decimated equalization filter designed in the frequency domain for the low-frequency filter of the crossover, $H_d(z)$ is a band-limiting filter to avoid aliasing due to the decimation by factor L, and $\tilde{H}(z)$ is the equalization filter for the high-frequency driver. The delay line of τ samples synchronizes the two signal paths in Fig. 24.

Mäkivirta et al. [123] proposed a model for loudspeaker characterization and a group-delay equalization procedure

taking into consideration a three-way loudspeaker with its crossover network. Experiments demonstrated that group-delay equalization at mid and high frequencies reduces the length of the loudspeaker impulse response without introducing pre-ringing [123].

5.2 Single Equalizer Before Crossover

Loudspeaker equalization can be separately applied to correct the response of the entire loudspeaker by pre-processing the input signal before it enters the crossover, according to Fig. 22(b). This is easier in practice than the per-band equalization, because there is no need to process the band signals, only the input signal, which is common to all bands. A target response must be defined, which is usually a high-pass filter to avoid distorting and damaging the low-frequency driver. [124].

The magnitude equalization is often implemented using graphic [125–128] or parametric equalizers [121, 129] that are manually or automatically adjusted to approximate a desired target curve. Ramos and Lopez [130] calculate the error areas between the frequency response of the loudspeaker and target curve, and Behrends et al. [129] minimize a cost function finding peak filters that best equalize the loudspeaker to the desired target frequency response. Similarly, Vairetti et al. [131] define a minimization function as the sum of square errors between the system and the target complex frequency responses, instead of the commonly used difference in magnitudes.

The use of warped FIR and IIR filters for loudspeaker equalization with the aim of increasing the approximation at low frequencies was proposed by Karjalainen et al. [132, 133] and became rapidly a popular method for loudspeaker equalization [134–136]. Ramos et al. [137] have introduced the use of a warped FIR filter with a cascade connection of parametric filters. Bank [138–140] proposed fixed-pole parallel second-order filters, for which a pole selection method based on warped IIR filter design shows a good performance for loudspeaker equalization. The fixed-pole method is conceptually easier and computationally cheaper than warped FIR filtering [140].

Furthermore, phase equalization is sometimes applied to loudspeakers, especially in the low-frequency range [124, 141]. It could be realized together with the magnitude equalization by applying a complex function or separately, exploiting all-pass filtering. Greenfield and Hawksford [142] proposed to use a minimum-phase IIR filter for magnitude equalization and to linearize the excess phase using a sampled time-reversed target impulse response. Herzog

and Hilsamer [143] proposed a method based on group-delay estimation at low frequencies that can be equalized using an all-pass filter exploiting time-reversal technique thus producing a long processing delay. Li et al. [144] proposed a non-minimum phase loudspeaker equalizer based on the design of IIR filters exploiting a balanced model truncation method. This method allows for defining the order of the filter during its application and obtaining both the minimum-phase component and the excess-phase correction with a stable IIR filter.

6 PERCEPTUAL STUDIES ON CROSSOVER FILTERS

The perceptual evaluation of the performance of crossover networks allows exploring the audible errors that occur during the reproduction in realistic conditions [145, 146, 32, 106, 147–149]. These errors can be caused by magnitude-response deviations or by phase distortion. Greiner [61] emphasized the difficulty of evaluating a crossover network in terms of the musical waveform. Following this idea, he applied a tone-burst test to obtain objective results that reflect the actual listening experience.

The first perceptual evaluation of a crossover network was performed by Wilson et al. [32] in 1989, employing a two-way FIR crossover. Listening tests were performed to evaluate the off-axis listening experience in which notches can be introduced in the response. The results showed that almost none of the subjects had perceived valuable differences [32]. Around the same time, Aarts conducted subjective tests with a new design method for crossover networks [146]. The experiments aimed at proving the similarity between a reference loudspeaker system and its equivalent experimental system.

Korhola and Karjalainen [106] introduced the just-noticeable differences (JNDs) to evaluate the perceptual performance when magnitude or phase errors occur in linear-phase FIR and Linkwitz-Riley IIR crossover filters. Regarding the magnitude errors, JNDs are observed when the deviation is about 1 dB [106]. Instead, it is more difficult to know when phase errors are perceived. In general, the phase distortion is more audible with headphones and when impulsive sounds are involved [106, 145]. Auditory analysis [106] demonstrated that, in most cases, signal degradations are not perceived using Linkwitz-Riley filters of order up to eight or using FIR filters of order up to 600.

Moreover, Møller et al. have found that the JND limit of group delay deviation is 1.6 ms[150]. This means that the errors are perceived when the group-delay change exceeds the JND limit. However, Liski et al. [39] have recently proved that even smaller group-delay variations of 0.6 ms may be audible in the frequency range between 500 Hz and 4 kHz in synthetic clicks.

Perceptual studies were also conducted by Dukic et al. [148, 149] to learn about the influence of the impulse response length of the crossover filters on perceived sound quality. In particular, the audibility of phase distortion introduced by magnitude complementary IIR crossover filters was compared to the magnitude complementary linear-phase FIR crossover networks through listening tests.

For IIR filters, Dukic et al. [148, 149] realized the crossover as a tapped cascaded interconnection of two all-pass subfilters [151], whereas for the FIR crossover, a pair of complementary filters was used. Two types of listening tests were conducted: the first involved the use of a pair of headphones using the sum of the output of the crossover filters as test signal; the latter considered a pair of loudspeakers in which the low-pass and high-pass–filtered signals were fed to the low-frequency and high-frequency drivers [148]. Several experiments were carried out using different music genres and different FIR and IIR crossover networks.

The results of Dukic et al. showed that the influence of the phase distortion and transition bandwidth on the perceptual evaluation depended on the involved subject and on the audio material [148]. In general, there was no major difference between the results for headphone and loudspeaker reproduction. Moreover, although the IIR filters have the largest phase distortion, they produce very subtle effects on the perceived sound quality and they exhibit similar results to the FIR filters. Therefore, the values of phase distortion and transition bandwidth are not significant. However, the IIR crossovers are a suitable choice because they exhibit a narrow transition band and amplitude linearity, as FIR filters, but with reduced computational complexity.

7 OTHER APPLICATIONS OF CROSSOVER FILTERS

Crossover filters have also found several uses in audio technology outside conventional loudspeaker systems. Recently, modal crossover networks have become appealing for the design of flat-panel loudspeakers [152, 153]. In this application, a crossover network is applied to an array of drivers distributed over a panel surface, allowing each band to be formed using an independent set of modes. Heilemann et al. have also applied crossover filters to divide the audio signal to low, mid, and high-frequency bands for a flat-panel display [154].

Another application area is multichannel sound reproduction, for example the adjustment of the low-frequency room correction using special satellite subwoofers with controllable directivity, as suggested by Hill and Hawksford [155]. Bharitkar and Kyriakakis have studied the selection of the crossover frequency and bass management filters for multichannel sound systems [156, 157]. Crossover filters have also a role in the control of satellite speakers and subwoofers in the NASA Exterior Effects Room, which allows the 3D reproduction of recorded aircraft flyover sounds [158]. Splitting of different Ambisonic orders to appropriate frequency bands in a 3D sound reproduction with room-reflection compensations has been tested using crossover filters [159]. Winter et al. have implemented the wave field synthesis by running the full system only at low frequencies and a local approximation at high frequencies, combining the two bands with a crossover network [160].

Multiband compression, used commonly for audio signal processing [40, 162–164], hearing aids [161, 165], and
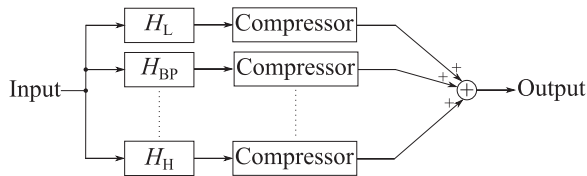
Fig. 25. Multiband compressor structure [161].

Fig. 26. Classification of crossover networks.

speech enhancement [166], is based on a band-splitting technique, which is similar to crossover filtering, as shown in Fig. 25. Also in these systems, it is desired that the overall magnitude response is flat, when the processing is deactivated. It has been shown that a two-band compressor already yields an improvement in speech discrimination in noise when compared to a single-band compressor [167, 168]. A multiband compressor can improve the coverage in radio broadcasting [169]. In a variation of this idea, Sang and Chen proposed a multiband expander and noise-gate with improved performance at low frequencies [170].

In many audio processing algorithms, crossover filters can help select the frequency band to which the operation is targeted. Fernándes-Cid and Casajús-Quirós [171] extended audio effects processing by applying a filterbank, not different from a multiway crossover network, and then suggested implementing a version of the same algorithm with different parameters for each band. Serrano demonstrated audio distortion effects on sub-bands [172]. Cho et al. suggested a sub-band switching technique for automatic musical tempo detection, also based on a multiway crossover network [173]. Parker [174] included a two-way crossover in his spring reverb algorithm to separate the low-frequency behavior, which could be down-sampled for computational savings. Several studies have improved acoustic crosstalk cancellation using crossover filters [175–177].

In computational room modeling, in which low frequencies are handled using the wave principle whereas high-frequency modeling uses the ray-tracing principles, a crossover network can help combine the low and high frequencies [178, 179]. In another room modeling application by Ouellet et al., the frequency-dependent properties of acoustic propagation and absorption vary for each octave band, and the band-splitting is implemented using a Linkwitz-Riley filterbank [180].

Crossover networks have similarities to filterbanks. This looks clear in early sub-band audio codecs, in which the input signal is divided into a few bands by a bank of filters, which does not differ from a crossover network [181]. A graphic equalizer [182, 183, 124, 184] is a specific filterbank, which has design questions similar to those of a crossover network. The main difference is that in a graphic equalizer, the band separation is usually between 20 and 40 dB, because the main function is to modify the spectral balance, whereas a crossover network must minimize the band leakage, precisely segregating each band into its own signal, which is consequently processed and played separately. In a hybrid FIR/IIR equalizer, Wang et al. applied crossover filters to divide the low frequencies to a warped IIR equalizer and the other frequencies to an FIR filter [134].
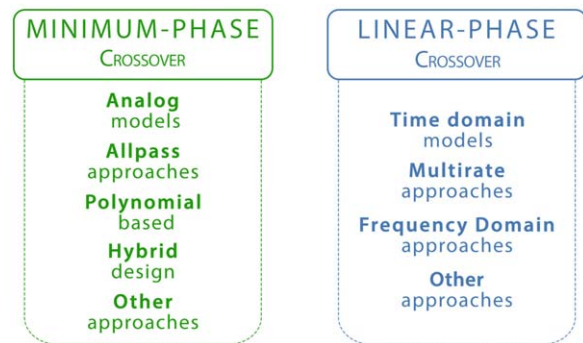
8 CLASSIFICATION AND COMPARISON

The categories of crossover filters are summarized in Fig. 26. Starting from a main division based on minimum-phase and linear-phase structure, several filter design methodologies are listed for each category. Table 4 summarizes all the approaches previously analyzed taking into consideration the network requirements listed at the beginning of SEC. 2. The computational cost and total latency are not considered here. It is evident that it is very difficult to fulfill the four requirements in the same system. The priority is the fulfillment of the magnitude flatness and the cutoff rate (requirements I and II) and then the linearity of the phase response (requirement IV), whereas the polar response behavior (requirement III) remains an aspect considered only in about 60% of the methods.

The magnitude flatness is generally achieved by designing the high-pass filter with a subtractive method or with methods that follow all-pass complementary conditions. Regarding the requirement of adequate steep cutoff rates of the individual filters in their stop-bands, not all the IIR methods shown in Table 4 verify the requirement II. The steepness of the transition band depends on the order of the filter. For example, in the case of Butterwoth or Linkwitz-Riley methods, a fourth-order filter can satisfy the requirement II, achieving a cutoff rate of about 24 dB/octave [37].

For the polar response, the on-axis response is usually considered; however, it is also desirable to have a good off-axis response. This was preliminarily studied by Greenfield [110] and by Lipshitz and Vanderkooy [73], who introduced the lobing error function to allow the comparison of crossover filters in terms of polar response. To eliminate the lobing error, D'Appolito [185] proposed a special arrangement for two-way systems. Later, Horbach and Keele [113] discussed how to design a three-way crossover to improve the off-axis response of non-coincident drivers. Hawksford [111] discussed the possibility of employing a random vector in the crossover filter response as a means of reducing the subjective significance of polar response errors. More recently, Catalá [47] presented a method based on the alignment at different angles within a listening window to obtain a better radiation pattern, avoiding off-axis cancellation.

Table 4. Overview of crossover filter designs taking into account the four main requirements of each method. Analog methods are specified with "Analog" in the column of the filter type.

Method	Filter type	Requirements				Method description
		I Flat	II Steep	III Polar	IV Phase	
Odd-order Butterworth [53]	Analog	✓	✓		✓	Parallel crossover, cf. Fig. 4(a)
Even-order Butterworth [65]	Analog		✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Linkwitz-Riley [19]	Analog	✓	✓	✓	✓	Parallel crossover, cf. Fig. 6
Bessel [89]	Analog		✓	✓		Parallel crossover, cf. Fig. 4(a)
Catalá [47]	Analog	✓	✓	✓		Parallel crossover, cf. Fig. 4(a)
Lipshitz and Vanderkooy [40]	Analog	✓	✓	✓	✓	Subtractive crossover, cf. Fig. 13(a)
Kyono and Fujiwara [77]	Analog	✓		✓	✓	Subtractive crossover, cf. Fig. 13(a)
D'Appolito [64]	Analog	✓			✓	Subtractive crossover, cf. Fig. 13(a)
Miller [90]	IIR	✓	✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Zhang et al. [91]	IIR	✓	✓			Parallel crossover, cf. Fig. 4(a)
Chutchavong et al. [58]	IIR	✓	✓		✓	Parallel crossover, cf. Fig. 4(a)
Hlurprasert et al. [92]	IIR	✓			✓	Parallel crossover, cf. Fig. 4(a)
Reviriego et al. [33]	IIR	✓	✓	✓	✓	All-pass–based crossover, cf. Fig. 15(a)
Wilson et al.[32]	FIR	✓	✓	✓	✓	Subtractive crossover, cf. Fig. 19
Schuck and Klowak [105]	FIR	✓	✓		✓	Subtractive crossover, cf. Fig. 23
Bruschi et al. [116]	FIR	✓	✓	✓	✓	Subtractive crossover, cf. Fig. 19
Azizi et al. [109]	FIR	✓	✓		✓	Parallel crossover, cf. Fig. 4(a)
Greenfield [110]	FIR	✓	✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Hawksford [111]	FIR	✓	✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Horbach [113]	FIR	✓	✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Haddad et al. [114]	FIR	✓	✓		✓	Parallel crossover, cf. Fig. 4(a)
Stanciu et al. [115]	FIR	✓	✓		✓	Parallel crossover, cf. Fig. 4(a)
Kyono and Fujiwara [77]	FIR	✓	✓	✓	✓	Parallel crossover, cf. Fig. 4(a)
Palestini et al. [95]	Hybrid	✓	✓			Subtractive crossover, cf. Fig. 17

Moreover, several studies have investigated the polar pattern of coaxial loudspeakers, in which the drivers share the same acoustic center [88, 186–190]. The co-axial design guarantees a flat magnitude response in the crossover region also in the off-axis positions [190], even if the speaker suffers from diffraction that cannot be corrected by the crossover. However, optimization techniques have been applied for co-axial loudspeakers to provide digital control of the radiated sound field, depending on the type of crossover network, directivity index, and polar pattern [187–189]. A linear-phase behavior can be achieved by using symmetric FIR filters, whereas it is not that easily achievable with IIR filters. What can be obtained is a quasi-linear behavior introducing all-pass filtering [40, 77].

Table 5 compares the previously described IIR and FIR crossover methods from a computational viewpoint. Analog filters have been converted into IIR digital filters through the bilinear transformation of Eq. (12). The computational cost is derived by considering a practical implementation of the resulting digital IIR and FIR filters. A numeric example is reported for a two-way crossover, using second-order IIR filters ($N_I = 2$) and FIR filters of order $N_F = 8$, where the order has been selected to provide the same cut-off rate as IIR filters with the 3-kHz crossover frequency.

Expectedly, the FIR filters have a higher computational load than IIR filters, as shown in Table 5. In fact, focusing on parallel crossovers, the FIR structure of Hawksford [111] shows 16 additions and 18 multiplications per output sample, whereas the IIR structure of Chutchavong [58] needs only eight additions and ten multiplications, which is 47% less. The subtractive configurations of Lipshitz [40] and Wilson [32], which are of IIR and FIR type, respectively, allow for a reduction of the computational complexity for both IIR and FIR methods by almost 50% with respect to the parallel structures. The all-pass–based crossover of Reviriego [120] shows the lowest computational complexity (four additions and five multiplications). In fact, the global two-way system of Fig. 15(a) has the same complexity of a single filter of order $N_I + (N_I - 2)/4$, where N_I is even [120]. Two examples of second-order Butterworth [65] and Linkwitz-Riley [19] crossover filters shown in Eqs. (13) and (17), respectively, are also included in Table 5, and have some of the lowest computational costs with six additions and four multiplications.

Analyzing the results shown in Tables 4 and 5, the all-pass–based crossover network proposed by Reviriego et al. [33] is the best solution, because it verifies all the requirements with the lowest computational cost. The same structure, shown in Fig. 15(a), was used by Harris et al. [71] in 2013. However, this system is a minimum-phase approach, so the phase linearity is not guaranteed for all the frequencies. Among the linear-phase approaches, the subtractive crossover by Wilson et al. [32] still seems to be the best solution, satisfying all the requirements with the lowest computational load. However, it is worth noting that the order of the employed FIR filters may increase with stricter specifications of the filter design and that a multi-way derivation further increases the latency of the system due to the cascade structure.

Table 5. Summary of digital crossover filter designs taking into account the computational complexity per output sample, when M is the number of bands of the crossover network, N_I is the IIR filter order, and N_F is the FIR filter order. A numerical example of the operation counts is given for a two-way crossover network ($M = 2$), second-order IIR filters ($N_\mathrm{I} = 2$), and order $N_\mathrm{F} = 8$ for the FIR filters.

Method	Description	Additions	Multiplications	Two-way crossover ($M = 2$, $N_\mathrm{I} = 2$, $N_\mathrm{F} = 8$)	
				N° Add.	N° Mult.
Butterworth [65]	Parallel IIR crossover cf. Fig. 15	$3N_\mathrm{I}$	$2N_\mathrm{I}$	6	4
Linkwitz-Riley [19]	Parallel IIR crossover cf. Fig. 6	$3N_\mathrm{I}$	$2N_\mathrm{I}$	6	4
Reviriego et al. [33]	All-pass–based crossover, cf. Fig. 15(a)	$5N_\mathrm{I}/2 - 1$	$5N_\mathrm{I}/2$	4	5
Lipshitz [40]	Subtractive IIR crossover, cf. Fig. 13(a)	$(M - 1)(2N_\mathrm{I} + 1)$	$(M - 1)(2N_\mathrm{I} + 1)$	5	5
Chutchavong [58]	Parallel IIR crossover, cf. Fig. 4(a)	$M(2N_\mathrm{I})$	$M(2N_\mathrm{I} + 1)$	8	10
Wilson et al. [32]	Subtractive FIR crossover, cf. Fig. 19	$(M - 1)(N_\mathrm{F} + 1)$	$(M - 1)(N_\mathrm{F} + 1)$	9	9
Hawksford [111]	Parallel FIR crossover, cf. Fig. 4(a)	MN_F	$M(N_\mathrm{F} + 1)$	16	18

9 CONCLUSION

A review on crossover networks has been presented. There are four main requirements for good crossover networks. Requirement I, the magnitude response flatness, within small tolerances, is always imposed in filter design. To fulfill requirement II, an adequately steep cutoff rate, can be achieved when the filter order is sufficiently high. Requirement III is related to uniform polar behavior in a loudspeaker system with non-coincident drivers. Finally, requirement IV favors a linear phase response, which can be achieved with digital FIR crossover filters, by introducing all-pass filtering to obtain a quasi-linear-phase behavior, or with low-order analog filters not producing a very nonlinear phase response. Especially in the case of digital networks, it is also important to consider the computational complexity and the total latency caused by crossover filtering.

The crossover network design methods can be classified into two main categories: minimum-phase and linear-phase approaches. In addition to analog filters, crossover filters can also be designed as digital IIR or FIR filters. IIR filters ensure a low computational complexity, whereas FIR filters can guarantee a linear phase response. Minimum-phase digital crossover filters can be categorized into basic approaches that are derived from the analog Butterworth or Linkwitz-Riley filters, all-pass–based networks, polynomial-based approaches, and hybrid approaches combining IIR and FIR digital filters. Linear-phase crossover networks can be mainly categorized into time-domain, multi-rate, and frequency-domain designs, which lead to FIR filter solutions.

Many approaches presented in the literature were compared in terms of the requirements and considering the computational complexity when all methods were converted to digital filters. The most efficient crossover network design fulfilling all requirements is the all-pass-filter–based Linkwitz-Riley IIR crossover network design, which has minimum-phase filters. The most efficient linear-phase FIR crossover satisfying each requirement uses complementary optimized FIR filters.

10 ACKNOWLEDGMENT

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