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Loudspeaker Damping

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Part 2. A discussion of theoretical considerations of loudspeaker characteristics, together with a practical method of determining the constants of the unit as a preliminary step in obtaining satisfactory performance.

WE NOW COME to the question of damping of the loudspeaker mechanism by the electrical circuit. In Fig 3 is shown the electrical equivalent of a loudspeaker illustrated in Fig. 2, with the addition of an electrical source of internal resistance R_G feeding it. This normally represents the R_p of the output tube or tubes as viewed from the secondary terminals of the output transformer.

The apparent generated voltage as viewed from the secondary terminals is e_G . The transient solution, however, is that current which flows in the network when e_G is zero, and subject to whatever initial conditions we seek to impose.

This circuit has been solved innumerable times; the current flow is oscillatory in nature, and of a frequency and decrement determined by the L , C , and R of the circuit. In particular, if

$$R = \sqrt{L_{me}/C_{me}}$$

$$= \frac{1}{2\pi f_r C_{me}} \quad (10)$$

where f_r is given by Eq. (8), and R is the resistance paralleling L_{me} and C_{me} , then the circuit is critically damped. This means that the natural frequency is zero, or the circuit is no longer oscillatory; physically the loudspeaker has no hangover effect. Of course R can be less than the value given by Eq. (10); the latter merely gives the maximum permissible value of R .

An inspection of Fig. 3. indicates that R must represent R_{me} paralleled by $(R_{vc} + R_G)$, hence if R_{me} is greater than the value required by Eq. (10),

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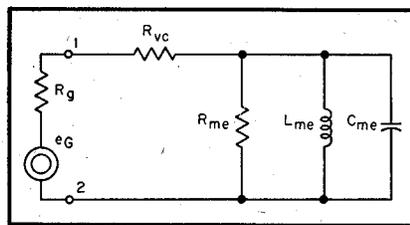


Fig. 3. Circuit of Fig. 2 with addition of generator.

$(R_{vc} + R_G)$ must be a low enough shunt to provide in conjunction with R_{me} the critical damping necessary.

It will be recalled from Eq. (5) that if the mechanical damping $(R_s + R_a)$ is low, R_{me} will be correspondingly high. An example which is to follow will show that usually the mechanical damping $(R_s + R_a)$ is very low, so that it can be expected that R_{me} will be relatively very high; much higher than will provide critical damping.

From this it follows that $(R_{vc} + R_G)$ must be a sufficiently low shunt to satisfy the critical damping condition given by Eq. (10). However, it is possible that the voice coil resistance R_{vc} is itself so high that Eq. (10) cannot be satisfied. In the usual case R_{vc} is not too high, but the maximum value left for R_G to assume can be quite low. In such a case a large amount of inverse voltage feedback may be necessary to reduce the source impedance to the requisite low value.

Numerical Example

The following numerical example will serve to illustrate the above analysis. Suppose we take a 16-inch cone type loudspeaker, whose mass is 40 grams, plus 4 grams for the voice coil. Assume further that the compliance of the suspension is $C_s = 3.2 \times 10^{-7}$ cm/dyne, and that the mechanical resistance is 2400 mechanical ohms.

To the mass of the cone and voice coil must be added that of the mass of the air. In the neighborhood of 25 cps or so, Olson² gives the reactance of the air load as 7500 mechanical ohms. The corresponding mass is

$$M_a = \frac{7500}{2\pi \times 25} = 48 \text{ grams}$$

Hence the total mass is

$$M_t = 40 + 4 + 48 = 92 \text{ grams}$$

The resonant frequency is, by Eq. (8)

$$f_r = \frac{1}{2\pi \cdot 92 \times 3.2 \times 10^{-7}} = 29.3 \text{ cps}$$

which is close to the value of 25 cps initially used to calculate the air mass.

The air also imposes a certain amount of damping in the form of radiation resistance. This is a rapidly varying function of frequency; from Olson's book we find it to be 600 mechanical ohms at 29 cps. Hence the total mechanical damping is

$$R_s + R_a = 2400 + 600 = 3000 \text{ mech. ohms.}$$

Now suppose the flux density B is 10,000 gauss, and the length l of voice coil conductor is 1500 cm. Assume further that the voice coil resistance R_{vc} is 10 ohms.

Then, from Eq. (5), we have

$$R_{me} = \frac{(1500 \times 10^4)^2 \times 10^{-9}}{3000} = 75 \text{ ohms}$$

$$C_{me} = \frac{92}{(1500 \times 10^4)^2 \times 10^{-9}} = 409 \mu f$$

$$L_{me} = (3.2 \times 10^{-7}) (1500 \times 10^4)^2 \times 10^{-9} = 0.072 \text{ henry}$$

Observe how large C_{me} is even though the mass responsible for this capacitive effect is only 92 grams.

For critical damping, the total resist-

² Loc. cit.

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ance shunting L_{me} and C_{me} must be, by Eq. (10):

$$R = \sqrt{\frac{.072}{409 \times 10^{-6}}} = 13.3 \text{ ohms}$$

Since R_{me} is one branch in parallel with R_{vo} plus the generator resistance, and this all totals 13.3 ohms, the voice coil branch must be

$$R_e = \frac{R_{me} \times R}{R_{me} - R} = \frac{75 \times 13.3}{75 - 13.3} = 16.18 \text{ ohms}$$

Since the voice coil resistance R_{vo} is 10 ohms, the generator or source resistance, as viewed from the secondary terminals of the output transformer, must be

$$R_G = 16.18 - 10 = 6.18 \text{ ohms.}$$

Although this is a low value, it is by no means prohibitively low. For example, if in the case of a single-ended triode output stage, $R_L = 2R_p$, then at the secondary terminals R_p should reflect as half of the voice coil load, if R_{vo} is 10 ohms, the reflected tube resistance R_G would be $10/2 = 5$ ohms. In short, a triode tube may be expected to act as critical damping in conjunction with the voice coil resistance.

In the case of a pentode tube, R_p is so high that no damping can be expected from it unless inverse voltage feedback is employed to an extent sufficient to lower the apparent source resistance to the required degree.

However, note that all this depends upon how low R_{vo} is compared to the length of wire used, and also how high the flux density B is. If the product (Bl) is low, both R_{me} and R may come out so low that R_{vo} alone may be in excess of that which paralleling R_{me} , will give the required value of R for critical damping. This means that even if the source resistance is zero, R_{vo} is too large and will not permit critical damping to be obtained.

Experimental Determination of Circuit Constants

It is possible to measure the motional impedance by simple electrical means, and from these measurements to determine the critical damping required. Since the measurements are to be made at the very low audio frequencies, ordinary iron vane meters can be used if so desired, and even a d.c. measurement of the voice coil resistance should be sufficient to furnish the value of R_{vo} .

If, however, it is desired to determine this quantity at the resonant frequency of the cone, or at any rate at some a.c. frequency, then the cone should be clamped so that it does not vibrate and generate a c.e.m.f., thereby furnishing a motional impedance value.

To measure the motional impedance, a set-up such as that indicated in Fig. 4 can be used. The audio oscillator wave

shape should be reasonably free of harmonics, and the audio amplifier should be capable of furnishing several watts of power without distorting. The ammeter can be of the iron-vane type, and should read one ampere or less at full scale. The voltmeter is preferably of a high-impedance type. A preliminary run should be made to determine the resonant frequency of the cone and its suspension. This is done by varying the frequency upward in steps starting from say, 20 cps, and noting E and I at each step. Their quotient is the impedance seen looking into the voice coil. This should be done with the field fully energized if it is of the electrodynamic type.

At the mechanical series resonant frequency of the cone, I will drop to a very low value, and E will tend to rise. In short, the quotient will be relatively

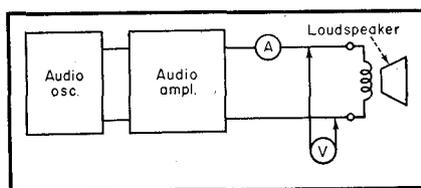


Fig. 4. Circuit arrangement used for making measurement of motional impedance.

large, and will represent $(R_{vo} + R_{me})$. If the value found previously for R_{vo} is subtracted from this reading, R_{me} is obtained. The resonant peak is normally quite sharp for reasons that will be explained further on.

In order to determine the value of critical damping R , it would appear necessary to measure L_{me} and C_{me} . However, R can also be determined by measuring the Q of the circuit; critical damping is obtained if $Q = 1$. To measure Q , ordinarily one merely has to plot the selectivity curve for the device, whether this curve represents transmission, impedance, admittance, or whatever other quantity gives this characteristic.

In the case of the loudspeaker, the resonant Q of the circuit is determined by the impedance as measured across R_{me} , L_{me} , and C_{me} in Fig. 3, with the electrical resistance $(R_{vo} + R_G)$ in parallel with R_{me} . In other words, the condition given by Eq. (10) for critical damping is also the condition for the resonant Q to be unity, where Q_r is in general determined by $\omega_r C_{me}$ and R_{me} and $(R_{vo} + R_G)$ in parallel.

Unfortunately, measurements must be made at terminals 1-2 in Fig. 3, since there are no accessible terminals across Z_{me} . The resulting impedance, Z_t , represents R_{vo} in series with Z_{me} , that is— with R_{me} , C_{me} , and L_{me} all in parallel. To find the above-defined resonant Q therefore requires some preliminary analysis, which will be given below.

Experimentally, however, all one has to do is to measure the impedance Z_t at and around resonance over a range including frequencies at which Z_t drops to $1/\sqrt{2}$ of its value at resonance (where it has the maximum value $R_{vo} + R_{me}$). Then, knowing the two frequencies at which this occurs, as well as the resonant frequency f_r , Q can be calculated. Once Q is known, the necessary value of R can be found, and then the maximum permissible generator resistance R_G .

Let us therefore proceed to evaluate this impedance. The impedance looking to the right into terminals 1-2 of Fig. 3 can be calculated from the circuit elements shown. It is:

$$|Z_t| = \sqrt{\frac{1 + Q_r^2 (1 - \rho^2)^2}{\left(\frac{1}{R_{me} + R_{vo}}\right)^2 + \frac{Q_r^2}{R_{vo}^2} (1 - \rho^2)^2}} \quad (11)$$

where Q_r is the resonant Q of the circuit if terminals 1-2 of Fig. 3 were short-circuited; i.e.,

$$Q_r = \omega_r C_{me} R = R / \omega_r L_{me} \quad (12)$$

in which R represents R_{me} and R_{vo} in parallel, and ω_r is the resonant angular velocity of L_{me} and C_{me} . Furthermore,

$$\rho = f / f_r \quad (13)$$

where f is the frequency at which Z_t is being measured, and f_r is the resonant frequency; in short, ρ represents the fractional deviation from the resonant frequency.

In particular, if $\rho = 1$, ($f = f_r$), Eq. (11) reduces to

$$Z_t = R_{me} + R_{vo} \quad (14)$$

which is correct from an inspection of Fig. 3, since at the resonant frequency L_{me} and C_{me} form a negligibly high shunt impedance across R_{me} , so that Z_t becomes $R_{me} + R_{vo}$, as stated above.

Furthermore, if $\rho = 0$, ($f = 0$), or $\rho = \infty$, ($f = \infty$), Z_t becomes equal to R_{vo} alone, as is also clear from Fig. 3, since L_{me} is a short circuit across R_{me} at $f = 0$, and C_{me} is the short circuit at $f = \infty$.

If Eq. (11) is solved for Q_r in terms of the other variables, there is obtained:

$$Q = \frac{1}{(1 - \rho^2)} \sqrt{\frac{\left(1 - \frac{Z_t}{R_{me} + R_{vo}}\right)^2}{(Z_t / R_{vo})^2 - 1}} \quad (15)$$

Now suppose the frequency is varied, which is the same as saying ρ is varied until Z_t drops to $1/\sqrt{2}$ of its maximum value; i.e.,

$$Z_t = \frac{R_{me} + R_{vo}}{\sqrt{2}} \quad (16)$$

If this value is substituted in Eq. (15), together with the corresponding specific

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LOUDSPEAKER DAMPING

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value of ρ , call it ρ_1 , there is obtained:

$$Q_r = \left(\frac{1}{1 - \rho_1^2} \right) \sqrt{\left(\frac{R_{me} + R_{vc}}{R_{vc}} \right)^2 - 2} \quad (17)$$

If $\left(\frac{R_{me} + R_{vc}}{R_{vc}} \right)^2 \gg 2$, say twenty times two, then Eq. (17) simplifies to

$$Q_r = \left(\frac{1}{1 - \rho_1^2} \right) \left(\frac{R_{vc}}{R_{me} + R_{vc}} \right) \quad (18)$$

If ρ_1 is nearly unity, the difference between the actual frequency f_1 and the resonant frequency f_r is small; that is,

$$\Delta f_1 = f_r - f_1$$

or

$$\Delta f_1 = f_1 - f_r$$

(depending upon whether the excursion is below or above the resonant frequency) is small. This is usually the case, and under such conditions Eq. (18) can be rewritten as

$$Q_r = \left(\frac{f_r}{2\Delta f_1} \right) \left(\frac{R_{vc}}{R_{me} + R_{vc}} \right) \quad (19)$$

which can form the basis of our experimental procedure as well as Eq. (18) can. If we re-write Eq. (19) as follows:

$$\frac{2\Delta f_1}{f_r} = \left(\frac{1}{Q_r} \right) \left(\frac{R_{vc}}{R_{me} + R_{vc}} \right) \quad (20)$$

we recognize the form to be similar to that of the well-known resonance formula, in which the fractional bandwidth ($2\Delta f/f_r$) for the half-power points is the reciprocal of the resonant Q of the circuit. Eq. (20) shows that owing to the point in the circuit at which the measuring instruments are introduced, the fractional bandwidth is reduced by

a factor $R_{vc}/(R_{me} + R_{vc})$, which would not occur if the measurements could be made across the motional impedance component itself.

The significance of Eq. (20) is that even though Q_r for a loudspeaker system may be less than unity, the fractional bandwidth will nevertheless be quite small because of the reducing factor $R_{vc}/(R_{me} + R_{vc})$. This makes the measurements somewhat critical and requires a well-calibrated frequency scale on the audio oscillator.

To see how this all fits together, let us proceed with an experimental run. The first measurement is R_{vc} ; this is found to be 10 ohms. Then the test setup of Fig. 4 is connected to the loudspeaker and the frequency varied from say 20 to 50 cps.

At 29.3 cps the current is found to dip to a minimum value of 83.2 ma, and the voltmeter reads 7.07 volts. The impedance is resistive, and of a value

$$R_{vc} + R_{me} = 7.07 / .0832 = 85 \text{ ohms.}$$

$$\text{Hence } Z_t = R_{me} = 85 - 10 = 75 \text{ ohms.}$$

Now the frequency is varied above and below 29.3 cps to the point where Z_t drops to $85\sqrt{2} = 60.1$ ohms, as found by taking the ratio of the voltmeter to ammeter readings in exactly the same way as $(R_{vc} + R_{me})$ was calculated.

Suppose the frequency drops from

29.3 to 26.7 cps before $Z_t = 60.1$, and rises to 31.9 cps before this value is reached once more. Then $\Delta f_1 = 29.3 - 26.7 = 2.6$ cps, or $\Delta f_1 = 31.9 - 29.3 = 2.6$ cps, and

$$2\Delta f_1/f_1 = 2 \times 2.6/29.3 = 0.1776.$$

We can now use Eq. (19) to calculate Q_r . Thus

$$Q_r = \left(\frac{1}{0.1776} \right) \left(\frac{10}{85} \right) = 0.663.$$

This is the Q of the loudspeaker circuit if the source impedance R_G were zero. Since Q_r is less than unity, it can be raised to that figure by allowing R_G to be greater than zero. It remains to calculate this value.

We have, for a parallel resonant circuit such as in Fig. 3, that

$$Q = \omega_r C_{me} R \quad (21)$$

where R is the resistance shunting C_{me} and L_{me} (Fig. 3), and is therefore R_{me} in parallel with $(R_{vc} + R_G)$. However, in the measurement and calculation yielding Q_r , R_G is essentially zero, and R represents simply R_{me} and R_{vc} in parallel.

We seek a value R' , such that the Q is equal to unity; i.e.,

$$1 = \omega_r C_{me} R'$$

or

$$R' = 1/\omega_r C_{me} \quad (22)$$

Substituting from Eqs. (21) and (20) in Eq. (22), we obtain

$$R' = \frac{R}{Q_r} = \frac{R_{me} R_{vc}}{R_{me} - R_{vc}} \times \frac{2\Delta f_1}{f_r} \times \frac{R_{me} + R_{vc}}{R_{vc}} = \frac{2\Delta f_1}{f_r} R_{me} \quad (23)$$

This represents R_{me} paralleled by $(R_{vc} + R_G)$, hence

$$R_{vc} + R_G = \frac{R' R_{me}}{R_{me} - R'} \quad (24)$$

and

$$R_G = \frac{R' (R_{me} + R_{vc}) - R_{vc} R_{me}}{R_{me} - R'} \quad (25)$$

Hence let us finish our experimental determination of R_G . From Eq. (23) we can find R' . If we use the last form, we have

$$R' = \frac{2\Delta f_1}{f_r} R_{me} = (0.1776) (75) = 13.31$$

ohms and from Eq. (25) we obtain

$$R_G = \frac{(13.31) (85) - (10) (75)}{(75 - 13.31)} = 6.19 \text{ ohms}$$

which of course checks the previous computation from the values for the mechanical constants, since it is the same loudspeaker that we have under consideration.

An Alternative Viewpoint

It is possible to reflect the electrical

constants into the mechanical side of the circuit, and obtain an alternative viewpoint of the behavior of the system as a whole. The results, so far as the low-frequency resonance is concerned, are the same, as will be shown. There is, however, another advantage of this alternative point of view with regard to the acoustical design; it permits the designer to incorporate the electrical constants into the acoustical design with a corresponding improvement in the performance of the loudspeaker.

First, the design formulas will have to be presented. The electrical impedance of the source and the voice coil appears in the mechanical side of the system as follows:

$$Z_{em} = \frac{(Bl)^2 \times 10^{-9}}{Z_e} \quad (26)$$

where Z_{em} is the mechanical impedance equivalent to the actual electrical impedance Z_e , and B and l have the same significance as before.

The output stage and voice coil in series with it exhibit essentially an inductive and resistive impedance at the higher audio frequencies. The inductance is the leakage inductance of the output transformer, plus that of the voice coil, and the resistance is the apparent source resistance R_G as viewed from the secondary terminals of the output transformer, plus that of the voice coil.

Hence, set

$$Z_e = R_e + j\omega L_e \quad (27)$$

where $R_e = R_{vc} + R_G$ (see Fig. 3), and L_e is the inductance defined above, and which we have not heretofore taken into account. At the lower audio frequencies $j\omega L_e$ can be ignored, whereupon Z_e reduces to R_e .

However, if Eq. (27) be substituted in Eq. (26), and then numerator and denominator divided by $(Bl)^2 \times 10^{-9}$, as before, there is obtained:

$$Z_{em} = \frac{1}{\frac{R_e}{(Bl)^2 \times 10^{-9}} + \frac{j\omega L_e}{(Bl)^2 \times 10^{-9}}} \quad (28)$$

If we consider $R_e/(Bl)^2 \times 10^{-9}$ as a mechanical conductance G_{em} , so that its reciprocal R_{em} , a mechanical resistance, is given

$$R_{em} = 1/G_{em} = 1/[R_e/(Bl)^2 \times 10^{-9}] \quad (29)$$

and if we further consider $L_e/(Bl)^2 \times 10^{-9}$ as a mechanical compliance C_{em} , then we can write Eq. (28) as

$$Z_{em} = \frac{1}{(1/R_{em}) + j\omega C_{em}} \quad (30)$$

or the electrical resistance and inductance in series appear in the mechanical system as a mechanical resistance and compliance in parallel. Hence, the counterpart of Fig. 3 is that shown in Fig. 5: a constant-velocity mechanical generator (counterpart of a constant-voltage electrical generator) feeds the mechanical resistance R_{em} equivalent to the electrical resistance R_e , in parallel with the mechanical compliance C_{em} equivalent to the electrical inductance L_e , and the actual mechanical impedance

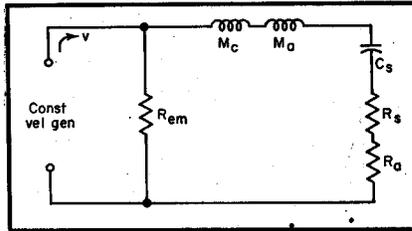


Fig. 5. Counterpart of Fig. 3, in mechanical terminology.

Z_m of the loudspeaker. This circuit has interesting implications both at the low- and at the high-frequency ends of the audio spectrum.

Consider the low-frequency end first. In this range C_{em} can be ignored, and

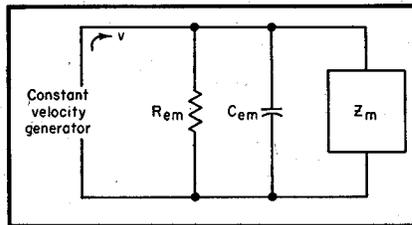


Fig. 6. Equivalent circuit corresponding to Fig. 5, showing damping due to electrical resistance.

Z_m consists, in the case of a direct-radiator cone loudspeaker, of the elements shown in Fig. 1. Hence Fig. 5 becomes circuit shown in Fig. 6. Here it is apparent how the electrical resistance R_e introduces in effect damping into the mechanical circuit by its transformed element R_{em} .

From Fig. 6 it is apparent that for critical damping,

$$\frac{(R_{em} + R_s + R_a)}{\sqrt{(M_c + M_a)/C_{em}}} = 1 \quad (31)$$

or alternatively, that the mechanical Q at resonance is unity:

$$Q_m = \frac{\omega_r(M_c + M_a)}{R_{em} + R_s + R_a} = 1 \quad (32)$$

from which the required electrical resistance must be

$$R_{em} = \omega_r(M_c + M_a) - (R_s + R_a) \quad (33)$$

Once R_{em} is evaluated from Eq. (33), the equivalent electrical resistance R_e

can be found from Eq. (29). Then the voice coil resistance R_{vc} is subtracted from R_e to yield the maximum permissible value of apparent generator resistance R_G .

Let us try out these formulas on the loudspeaker constants given previously. It will be recalled that the total mass (including that of the voice coil) was 92 mechanical ohms. This will be the value used for $(M_c + M_a)$. The resonant frequency was 29.3 cps, so that $\omega_r = 2\pi \times 29.3$ rad./sec. Also $(R_s + R_a)$ came out to be 3000 mechanical ohms.

Hence, if the appropriate values be substituted in Eq. (33), there is obtained:

$$R_{em} = (2\pi \times 29.3)(92) - (3000) = 16,980 - 3,000 = 13,980 \text{ mech. ohms}$$

Now, from Eq. (29), the equivalent electrical resistance R_e that is required to obtain critical damping is

$$R_e = \frac{(Bl)^2 \times 10^{-9}}{R_{em}} = \frac{(10000 \times 1500)^2}{13980}$$

$= 16.13$ ohms (electrical). Since the voice coil resistance R_{vc} is 10 ohms, the apparent source resistance can be

$$R_G = 16.13 - 10 = 6.13 \text{ ohms}$$

which checks our previous calculations, as it should.

High-Frequency Response

The same equivalence between circuits can be utilized in the analysis of a high-frequency tweeter unit of the horn type. This employs a small diaphragm and voice coil, which feeds the cavity in front of it that leads to an exponential horn. The physical arrangement is shown in cross-section in Fig. 7. Here m_d represents the mass of the diaphragm and associated voice coil; C_a , the compliance of the air chamber in front of the diaphragm, necessary to furnish clearance for the motion of the diaphragm and useful in building out the mechanical circuit; and finally r_h represents the acoustical resistance of the horn throat in the frequency range above its low-frequency cutoff point.

The mechanical circuit has been analyzed many times in the past; it is given in Fig. 8. The resistance r_h is that of the throat of the horn, and is equal to the area of the throat in sq. cm. multiplied by 41.4 mech. ohms, which is the radiation resistance of air per sq. cm. A_d is the area of the diaphragm; in conjunction with A_h it forms a kind of hydraulic press which is the mechanical counterpart of an electrical transformer. The step-down ratio is A_d to A_h ; conversely r_h is reflected to the diaphragm as an equivalent resistance r'_h such that

$$r'_h = (A_d/A_h)^2 \quad (34)$$

The reflected resistance r'_h shunts the air chamber compliance C_a . This is because the lower r'_h is, the more readily can it relieve the pressure built up in the air chamber by the motion of the diaphragm. This is exactly analogous to the reduction in the charge and voltage across a capacitor when it is shunted by a low resistance.

From Fig. 8 the loudspeaker unit is recognized as forming an L-section low-pass filter. For proper transmission up to the cut-off frequency, it is necessary that

$$r'_h = \sqrt{M_d/C_a} \quad (35)$$

The cutoff frequency is given by

$$f_c = \frac{1}{\pi\sqrt{M_d C_a}} \quad (36)$$

If twice the mass ($2M_d$) were employed and another compliance C_a placed at the left end, a π -section filter would be obtained, to which Eqs. (35) and (36) would apply equally well. In short, the same cutoff frequency can be obtained

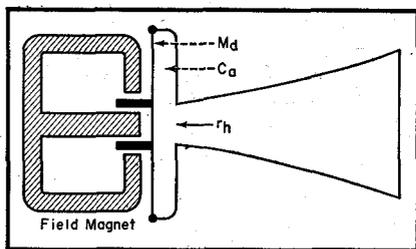


Fig. 7. Physical arrangement of mechanical elements of a high-frequency horn and unit.

for double the mass, if a compliance is placed at the other end of it.

If only the mass is doubled, then the cutoff frequency is reduced to 70.7 per cent of its original value, as is evident by substituting $2M_d$ for M_d in Eq. (36). (The corresponding changes in r'_h are not of importance as they involve merely a change in the ratio of A_d to A_h .)

For a given high-frequency cutoff and power-handling ability of the speaker, the diaphragm mass M_d comes out to be a certain amount. If M_d can be kept the same, and yet a compliance placed at the front end, the high-frequency cutoff can be extended to $\sqrt{2}$ or 1.414 times its original value without altering the speaker's power handling ability. Hence it is of interest to see how this can be done.

At the higher audio frequencies, the output transformer appears at its secondary terminals essentially as a series inductance L_L (its leakage inductance). The power amplifier tubes, as reflected to the secondary of the transformer appear as a resistance R_G in series with L_L . To this must be added the voice coil resistance R_{vc} and its inductance L_{vc} in

series with R_G and L_L . Hence finally the electrical current appears as

$$Z_e = R_e + j\omega L_e$$

where

$$R_e = R_G + R_{vc} \quad (37)$$

and

$$L_e = L_L + L_{vc}$$

Figure 5 and Eq. (30) show how these appear in the mechanical circuit. The mechanical impedance Z_m is in this case illustrated by Fig. 8, so that finally in Fig. 9 is given the complete mechanical circuit including the equivalent electrical circuit parameters.

Here, in accordance with Eq. (29)

$$R_{em} = \frac{1}{R_e/(Bl)^2 \times 10^{-9}} \quad (29)$$

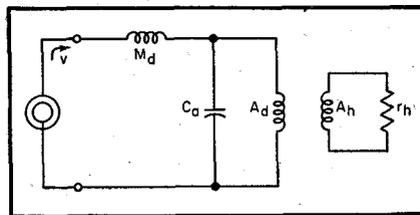


Fig. 8. Mechanical circuit of high-frequency speaker.

and this should match r'_h for maximum power transfer, or

$$R_e = (Bl)^2 \times 10^{-9} / r'_h \quad (38)$$

from which the apparent source impedance should equal

$$R_G = R_e - R_{vc} = \frac{(Bl)^2 \times 10^{-9}}{r'_h} - R_{vc} \quad (39)$$

The apparent mechanical compliance equivalent to the electrical inductance is indicated by Eqs. (28) and (30), namely:

$$C_{em} = L_e / (Bl)^2 \times 10^{-9} \quad (40)$$

However, in order to convert the L-section mechanical low-pass filter of Fig. 8 into the π -section low-pass filter of Fig. 9, it is necessary that

$$C_{em} = C_a = \frac{L_e}{(Bl)^2 \times 10^{-9}} \quad (41)$$

If such coordination in electrical and mechanical design be accomplished, a 41 per cent increase in frequency response may be expected over the case of no electrical inductance at all. Of course, in actual practice the electrical system inherently has inductance and resistance so that the "building-out" of the L-section into a π -section tends to take place; all that it is desired to point out here is that the electrical and mechanical circuit elements can be coordinated so as to improve the performance rather than to have a haphazard relationship to one another, and that furthermore, electrical inductance is not necessarily an undesirable characteristic in the output stage, but can serve a useful purpose.

Undoubtedly, in most systems the inductance—particularly that of the voice coil itself—is too high and produces a C_{em} in excess of C_a . Also, R_{em} may be too low compared to r'_h because of excessive electrical resistance $R_G + R_{vc}$. However, this serves to counterbalance an excessive value for C_{em} and therefore tends to smooth out the response.

The interested experimenter can calculate the actual response of the network shown in Fig. 9 on the basis that it is not a truly terminated low-pass filter section, since a resistance such as r'_h is but a nominal match over the pass

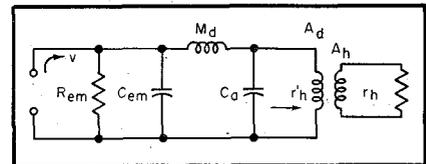


Fig. 9. Conversion of Fig. 8 circuit to pi-section equivalent.

band, and is a considerable mismatch near the cutoff frequency, where the termination should approach zero. He can also calculate the response for his actual speaker and amplifier output stage, in order to see directly the effect of varying, for example, the electrical circuit constants.

Conclusion

A method of coordinating the motional impedance of a loudspeaker with the electrical impedance has been presented here with the object of reducing "hangover" effects and objectionable transients in general at the low-frequency resonance of the speaker.

An experimental method has also been presented to enable the necessary measurements to be made in order that the correct source impedance be obtained for critical damping of the system. The method requires merely an audio oscillator, an a-c voltmeter and an a-c ammeter in order to determine the impedance over a range of frequencies. From the shape of the impedance curve the Q of the system can be determined, and from the value of voice coil and motional impedance at resonance, the requisite source resistance for critical damping can be calculated.

An alternative method based on viewing the electrical constants from the mechanical side was then presented, and it was shown that this method led to the same answers as above. Finally, it was shown by this method how inductance and even resistance in the electrical system could be put to use to obtain a coordinated system in the case of a high-frequency loudspeaker.